Dynamic Models, New Gains from Trade?*

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Abstract

Yes. We state closed-form expressions for steady state gains from trade that apply in a class of dynamic trade models that includes dynamic versions of the Krugman (1980), Melitz (2003), and customer capital (e.g., Arkolakis, 2010) models. As in Arkolakis, Costinot, and Rodríguez-Clare (2012), the gains are a function of the domestic trade share and the long-run elasticity of trade with respect to iceberg trade costs. This elasticity, however, cannot be recovered from the long-run tariff elasticity alone, because the dynamic trade adjustment mechanism present in this class of models increases the difference between the iceberg and the tariff elasticity in the long run. Our main substantive finding is that the dynamic gains from trade are unambiguously greater than those implied by static models, conditional on the same long-run tariff elasticity. We propose an approach to implement the formula by combining tariff elasticity estimates at multiple horizons and demonstrate that the difference relative to static models can be substantial. Accounting for the transition path has a modest impact on the magnitude of the gains from trade, relative to simply comparing steady states using the formula.

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1. Introduction

The last decade has seen a veritable explosion of work employing dynamic quantitative trade models. These models are useful for studying a number of salient mechanisms and phenomena, including the gradual adjustment of trade flows to trade cost shocks, the interaction between factor accumulation and trade, and the role of firms' forward-looking export entry decisions.[1] However, studying the determinants of the gains from trade in dynamic environments is often challenging. There are currently few analytical characterizations of the dynamic gains from trade, as these models are typically solved numerically and are often computationally intensive. In particular, we currently lack compact and intuitive gains from trade formulas in the spirit of Arkolakis, Costinot, and Rodríguez-Clare (2012, henceforth ACR) for dynamic economies.

This paper makes three contributions. Our theoretical contribution is to state closed-form expressions for the gains from trade (GFT) that apply in a class of dynamic models in steady state. The formula circumvents the need to solve computationally intensive dynamic trade models, and thus provides a useful benchmark for assessing the dynamic gains from trade. Similar to ACR, welfare gains from trade are a function of the domestic absorption share and the long-run iceberg trade elasticity. Different from ACR, the long-run iceberg elasticity is a function of multiple structural parameters, including a parameter that captures the difference between the long-run and short-run adjustments to trade cost shocks. Our measurement contribution is to show how elasticity estimates at different time horizons can be used to recover the structural parameters required to calculate the GFT in our dynamic setting. We show that conditional on the same long-run tariff elasticity, the dynamic GFT are strictly higher than those implied by the ACR formula. Lastly, we quantify the GFT. Under our preferred tariff elasticity estimates, the gains from trade in this class of models are large. Furthermore, the GFT can be large even under high values of the long-run tariff elasticity. We compute a multi-country general equilibrium dynamic trade model, and compare our formula-implied steady state gains to those that explicitly account for the transition path. Accounting for the transition has a modest impact on the GFT, regardless of whether countries have access to international bond markets.

Theory. To illustrate the model features that are important for our main theoretical result, we start with a simple dynamic Krugman (1980) model. There are multiple countries and firms. Firms face downward-sloping demand in destination markets and earn positive flow profits each period. In order to enter a destination market, a firm has to pay a stochastic sunk cost. Firms enter a market if the net present value of their expected profits from selling there covers the sunk costs of entry. These features introduce forward-looking behavior and gradual adjustment to shocks. Following a reduction in trade costs, two forces will act on the welfare of the domestic agents: the gain from imported varieties, captured by the domestic trade share as in ACR; and the loss of domestic varieties.

[1] Dynamic trade models have a tradition going back to at least the 1960s (e.g., Bardhan, 1965, 1966; Oniki and Uzawa, 1965; Inada, 1968; Stiglitz, 1970).

We find a tractable representation of the dynamic entry problem that yields a closed form solution for the mass of domestic varieties as a function of the domestic trade share in steady state. The tractability comes from a particular distributional assumption for stochastic sunk costs (inverse Pareto) and could potentially have broader applications. It turns out that under this distributional assumption, the mass of domestic varieties is a power function of the domestic trade share. Thus, the domestic trade share is a sufficient statistic for the welfare change, modulo the relevant elasticity. This elasticity is a function of the Dixit-Stiglitz substitution elasticity between firms' products σ, and the dispersion parameter of the sunk cost distribution χ. Intuitively, this parameter controls how strongly domestic variety responds to foreign competition.

We then state a general set of conditions under which the closed-form expression for the gains from trade applies. The first two conditions coincide with ACR: trade is balanced *in steady state*; and the ratio of aggregate profits to aggregate sales is constant. The third condition puts restrictions on supply and demand. Without loss of generality, total bilateral exports can be written as a product of the mass of firms (or customers) and sales per unit mass. The result requires that (i) domestic demand per unit mass of firms/customers has a constant elasticity of substitution functional form (closely related to ACR's third assumption), and (ii) the mass of firms/customers is a power function of sales per firm normalized by the source country wage. The latter condition differs from ACR, whose result obtains when this mass is constant. Qualitatively, it is intuitive that the mass of firms increases in per-firm sales relative to factor cost, as it follows from the logic of common entry problems. Sales relative to factor cost is a reflection of profits; when profits are higher, more firms enter. However, the power functional form of this relationship is a non-trivial restriction. In the Krugman example above it is micro-founded with a distributional assumption on sunk cost draws, but that is not the only possible microfoundation, as we show below.

Our main theoretical contribution is to derive a closed-form GFT formula in a general dynamic environment satisfying these assumptions. The steady state real consumption level under trade relative to autarky is given by

$$\lambda_{jj}^{\frac{1}{\varepsilon_{\kappa}^{0}(1+\chi)}}, \tag{1.1}$$

where λ_{jj} is the share of domestically-produced goods in total spending, ε_{κ}^{0} is the elasticity of the CES demand per unit mass to unit costs, and χ is the exponent governing the relationship between the mass of firms and per-firm sales.[2] To fix ideas, in the dynamic Krugman model, ε_{κ}^{0} is simply $1-\sigma$. Importantly, $\varepsilon_{\kappa}^{0}(1+\chi)$ is also the *long-run* elasticity of trade with respect to the iceberg trade costs.

A critical difference with respect to ACR is that the long-run trade elasticity is governed by multiple structural parameters. The reason is that in our setting, both the long-run trade elasticity and the GFT reflect an additional margin of adjustment. In the dynamic Krugman model, this additional margin

[2]The formula compares steady state consumption levels, and requires balanced trade only in steady state. Countries opening up to trade can borrow and lend internationally along the transition path.

is the changing mass of varieties over time and across different equilibria.[3] This margin is switched off in ACR, and thus both the trade elasticity and the formula's exponent reflect only the consumption gain from foreign varieties ($1/(1-\sigma)$ in the Krugman case). Indeed, we show that if the mass of domestic varieties were constant, our formula coincides with ACR. The welfare impact of the loss of domestic varieties scales with the domestic trade share with elasticity $-\frac{1}{1-\sigma}\frac{\chi}{1+\chi}$. The first term in this composite fraction reflects the welfare losses from lower domestic variety. The second captures the responsiveness of domestic variety to the changes in domestic profitability brought about by trade cost changes, and is governed by the parameter χ. The gains from foreign variety and losses from domestic variety add up to the overall elasticity of welfare with respect to the domestic trade share: $\frac{1}{1-\sigma} - \frac{1}{1-\sigma}\frac{\chi}{1+\chi} = \frac{1}{(1-\sigma)(1+\chi)}$ as in (1.1).

We show that the conditions for our theoretical result are satisfied by two additional dynamic models: (i) the customer base model à la Arkolakis (2010) with the cost of acquiring customers taking a power form and gradual customer base adjustment to trade cost shocks; and (ii) a dynamic version of the Melitz (2003) model with Pareto productivity and inverse Pareto sunk cost distributions, in which the set of firms selling to each destination adjusts gradually in response to shocks. We also develop an extension to include physical capital accumulation as in, e.g., Alvarez (2017) or Ravikumar, Santacreu, and Sposi (2019), and discuss how firm entry interacts with capital accumulation.

Measurement. While domestic trade shares are easy to obtain, the long-run elasticity of trade with respect to iceberg costs is not. The main reason is that we typically do not observe iceberg trade costs. Instead, the predominant approach in the literature is to use tariff variation, as tariffs are often the only directly observed component of *ad valorem* trade costs.[4] Thus, we proceed on the presumption that the researcher has an estimate of the long-run elasticity of trade flows to tariffs, denoted ε_τ, but requires the long-run elasticity of trade with respect to iceberg costs $\varepsilon_\kappa \equiv \varepsilon_\kappa^0 (1+\chi)$ to implement the GFT formula.

The distinction between the iceberg and the tariff elasticities is innocuous in many conventional static settings. In the class of models covered by ACR, the former can be easily recovered from the latter by adding 1: $\varepsilon_\kappa = \varepsilon_\tau + 1$. This is no longer the case in dynamic environments, which feature the additional adjustment margin discussed above. To make this point explicit, we state a generalization of the main proposition to an environment with both iceberg costs and tariffs. The GFT formula still requires the long-run elasticity of trade with respect to iceberg trade costs. However, in a dynamic world the long-run elasticity of trade with respect to iceberg trade costs cannot be recovered from the

[3] In the Krugman model, the mass of *potential* varieties and the mass of actually-produced varieties coincide. In other dynamic models that satisfy our assumptions, this additional margin of adjustment reflects different microfoundations. For example, in the Melitz case, it captures changes in the mass of potential (or "innovated") varieties, which is distinct from the mass of actually-produced varieties as not all potential varieties are produced in equilibrium. In the customer capital model, this margin captures changes in the mass of customers.

[4] At least 20 papers have used tariff variation to estimate the trade elasticity in the past 25 years. See Head and Mayer (2014) and Boehm, Levchenko, and Pandalai-Nayar (2023) for references.

long-run tariff elasticity alone. Instead, the two elasticities have the following relationship:

$$\varepsilon_\kappa = \varepsilon_\tau + 1 + \chi,$$

necessitating not only the long-run tariff elasticity, but also a value for χ.

A direct consequence is that conditional on the same long-run tariff elasticity ε_τ, the GFT implied by the dynamic formula are strictly larger than the GFT according to ACR. This is immediate from the fact that the ACR formula is $\lambda_{jj}^{\frac{1}{\varepsilon_\tau+1}}$, whereas our formula for dynamic models is

$$\lambda_{jj}^{\frac{1}{\varepsilon_\tau+1+\chi}},$$

and $\chi > 0$ as required by model microfoundations. Furthermore, holding ε_τ fixed the GFT are increasing in χ.

We propose a general and internally consistent way to recover χ from the data. This parameter can be inferred from the ratio of two tariff elasticities at different time horizons: the short- and the long-run. In the short run, the mass of export variety is predetermined, but in the long run it fully adjusts. Thus, the ratio of the long- to the short-run tariff elasticities reflects the magnitude of the dynamic adjustment of export variety, which is precisely what χ regulates.

Ideally, these tariff elasticities should be obtained from dynamic gravity equation estimates. Boehm, Levchenko, and Pandalai-Nayar (2023) show that all the different microfoundations covered by the theoretical results above generate dynamic gravity equations, which they then estimate. Thus, our proposed strategy of recovering the required elasticities from dynamic gravity estimates has the advantage of applying across multiple microfoundations. This approach is a dynamic generalization of the well-accepted practice of using gravity equations as a source of the trade elasticity for static models.

Quantification. We first report the dynamic gains from trade in steady state for a large set of countries according to our formula. Our preferred short- and long-run tariff elasticity estimates are taken from Boehm, Levchenko, and Pandalai-Nayar (2023). The GFT are substantial, with gains of 25-30% for even the largest and most closed countries such as the US, Brazil, and China, and gains of over 100% for several countries. Adjusting the formula for tariff revenues, along the lines of Felbermayr, Jung, and Larch (2015), plays a small role in all but a handful of economies.

Second, we show that the difference relative to the ACR formula can be large. For example, under our preferred calibration, the US GFT are 10.5% under ACR, but are 28.3% according to our formula, even though both are computed with the same ε_τ. Relatedly, it is well-known that the ACR formula implies modest GFT under larger values of the tariff elasticity (such as $\varepsilon_\tau = -5$ as in Costinot and Rodríguez-Clare, 2014). The static ACR formula requires low long-run tariff elasticities to yield large gains, a point explored in detail by Ossa (2015). By contrast, the dynamic formula can exhibit large

GFT even under high values of the long-run tariff elasticity such as -5. Large long-run tariff elasticities therefore do not necessarily imply small gains from trade in our dynamic class of models.

Third, we compute the GFT in a 30-country general equilibrium dynamic Krugman model, accounting for the transition path from one trade regime to another. In our baseline scenario, a single country transitions from autarky to trade or from trade to autarky. All countries have access to international bond markets. The length of the transition is disciplined in part by the time it takes for the trade elasticity to converge to its long-run value, which Boehm, Levchenko, and Pandalai-Nayar (2023) estimate to be around 7-10 years. We also consider a variety of additional scenarios, including cases in which all countries transition simultaneously and cases in which there is no trade in international bond markets. While interesting in their own right, these exercises also help evaluate the usefulness of the formula as an approximation for the GFT in computationally challenging dynamic models under a range of assumptions.

There are three main findings. First, the disparity between the formula-implied steady state welfare gains and the gains taking into account the transition path is modest, with a difference of about $9-15\%$, on average. Second, the steady state formula overstates the dynamic gains of the transition from autarky to trade, but understates the gains from staying open to trade compared to the transition from trade to autarky. The reason is as follows. In the opening scenario, the country begins in the autarky steady state and transitions to the trade steady state slowly. Over this transition, consumption is lower than the eventual steady state consumption, because firms are still setting up exporting operations. As a result, the dynamic gains of going from autarky to trade are below the steady state comparison. When moving from trade to autarky, countries' accumulated exporting capital has become useless. At the same time, the mass of domestic varieties is below the autarky steady state level and the country must accumulate domestic firms to replace imports. Thus, when shocked with an unanticipated increase in trade costs, countries temporarily decrease consumption below the level of the eventual autarky steady state. This reduces the value of the consumption path towards autarky – effectively the denominator of the GFT – relative to steady state, and thus raises the implied GFT. Finally, access to international bond markets during the transition and whether other countries are already open or everyone opens simultaneously have essentially no effect on the GFT over the full dynamic transition path. This suggests that the analytical formula has potentially wide applicability across different asset market structures and trade cost disturbances.

Literature. While the field of international trade has always been interested in the gains from trade, the literature on the quantification of GFT was given fresh impetus by the landmark contribution of Arkolakis, Costinot, and Rodríguez-Clare (2012), who stated closed-form expressions for the GFT in a wide class of static trade models.[5] This led to an active literature exploring various analytical and quantitative properties of the sufficient statistics formulas, such as sectoral comparative advantage

[5] Antecedents that stated similar formulas in specific settings include Eaton and Kortum (2002) for the Ricardian model, Eaton and Kortum (2005) for the Armington model, and Arkolakis et al. (2008) for the Melitz model.

(Costinot and Rodríguez-Clare, 2014; Levchenko and Zhang, 2014) or trade elasticities (Ossa, 2015; Imbs and Mejean, 2017). The formulas have also been extended in a variety of directions, such as variable markups (Arkolakis et al., 2019; Heid and Stähler, 2024), non-constant trade elasticities (Melitz and Redding, 2015; Feenstra, 2018; Adão, Arkolakis, and Ganapati, 2020), gains from multinational production (Ramondo and Rodríguez-Clare, 2013), non-representative agent settings (Galle, Rodríguez-Clare, and Yi, 2023; Waugh, 2023), and accounting for tariff revenue (Felbermayr, Jung, and Larch, 2015; Lashkaripour, 2021) and distortions (Bai, Jin, and Lu, 2024), to name a few. In static settings, Melitz and Redding (2015) and Feenstra and Weinstein (2017) highlight that allowing for changes in the mass of (potential) firms leads to welfare gains that differ from the ACR formula, implying that the GFT can then be sensitive to microfoundations. In our dynamic trade setting the mass of firms also changes, contributing to the gains from trade. Aside from the fact that ours is a dynamic setting, our theoretical contributions relative to these papers are to (i) analytically characterize the mapping between the mass of firms and the domestic trade share, yielding ACR-like GFT welfare formulas that account for endogenous mass adjustment; and (ii) establish this mapping in a class of models that covers multiple microfoundations.

The literature on analytical GFT characterizations in dynamic environments is more limited. Arkolakis, Eaton, and Kortum (2011) and Chen et al. (2025) develop results for dynamic versions of the Eaton-Kortum model, Atkeson and Burstein (2010) for dynamic heterogeneous firm models, and Fitzgerald (2025) for the Armington model. On the quantitative side, a number of papers numerically compute the GFT in dynamic models, including accounting for the transition path (see, among others, Alvarez, 2017; Brooks and Pujolas, 2018; Mutreja, Ravikumar, and Sposi, 2018; Ravikumar, Santacreu, and Sposi, 2019, 2024; Anderson, Larch, and Yotov, 2020; Alessandria, Choi, and Ruhl, 2021).[6] We provide a relatively general analytical characterization that applies to steady state comparisons in a broad class of dynamic trade models. Unlike the Armington or Eaton-Kortum settings, our analytical results cover cases in which there is net firm entry and profits. We also emphasize the importance of measurement, in particular the information contained in trade elasticities at multiple horizons in conditioning the steady state gains from trade.

The remainder of the paper is organized as follows. Section 2 lays out the simplest dynamic model to illustrate the mechanics behind the result. Section 3 states several general results and establishes the mappings to other dynamic models. Section 4 discusses the measurement of the elasticity required to implement the formula. Section 5 quantifies the gains from trade and Section 6 concludes.

[6] Beyond the focus on the GFT, the vast and constantly growing literature on dynamic trade cannot be comprehensively summarized here (see, among many others, Costantini and Melitz, 2007; Ruhl, 2008; Drozd and Nosal, 2012; Burstein and Melitz, 2013; Alessandria and Choi, 2014; Ruhl and Willis, 2017; Leibovici and Waugh, 2019; Alessandria, Arkolakis, and Ruhl, 2021; Alessandria, Choi, and Ruhl, 2021; Kleinman, Liu, and Redding, 2023; Steinberg, 2023; Blaum, 2024; Fitzgerald, Haller, and Yedid-Levi, 2024).

2. Warmup: Welfare Gains in a Dynamic Krugman Model

This section derives the GFT formula in the simplest possible setup: a dynamic version of the Krugman (1980) model. It serves to introduce the notation maintained throughout the paper, and to highlight the features essential for our main result.

2.1 Model Setup

Consider a dynamic economy with J countries indexed by i and j, and discrete time indexed by t. Each country is populated by a representative consumer who consumes C_{jt} and inelastically supplies labor L_j.

Households. Consumers in country j maximize

$$\max_{\{C_{jt}\}} \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\gamma}}{1-\gamma}$$

subject to the budget constraint

$$P_{jt}C_{jt} + B_{jt} + B_{jt}^{*} + w_{jt}\psi\left(\frac{B_{jt}^{*}}{P_{jt}}\right) = w_{jt}L_j + \Pi_{jt} + R_{jt}^{g} + B_{jt-1}\left(1 + r_{jt-1}^{n}\right) + B_{jt-1}^{*}\left(1 + r_{t-1}^{n*}\right), \tag{2.1}$$

and a no-Ponzi game condition. Here, P_{jt} is the consumption price index in country j, w_{jt} the nominal wage, Π_{jt} aggregate profits, and R_{jt}^{g} are government tariff revenues rebated to the household. Domestic bond holdings B_{jt}, satisfying $B_{jt} = 0$ in equilibrium, are included in the budget constraint only to price country j's nominal interest rate r_{jt}^{n}. The international bond B_{jt}^{*} is traded in equilibrium. It yields nominal interest rate r_t^{n*} and is subject to holding costs $\psi\left(B_{jt}^{*}/P_{jt}\right)$, denominated in units of domestic labor. We include these costs in the model to ensure that in any steady state bond holdings are zero and thus trade is balanced.[7] Outside of the steady state, however, countries will generally borrow or lend, and hence trade is generally not balanced. We assume that the bond holding cost function ψ satisfies $\psi(0) = 0$, $\psi'(0) = 0$, and $\psi''(0) = \psi$. We further assume that firms producing in country j are exclusively owned by the consumer in j, and hence the consumer receives all profits as income. The parameters β and γ denote the household's discount factor and the coefficient of relative risk aversion, respectively.

[7] See Schmitt-Grohé and Uribe (2003) for a discussion of bond holding costs.

Optimal behavior implies that consumption follows the Euler equations

$$C_{jt}^{-\gamma} = \left(1 + r_{jt}\right) \beta C_{jt+1}^{-\gamma},$$
$$C_{jt}^{-\gamma} = \frac{1 + r_{jt}^{*}}{1 + \frac{w_{jt}}{P_{jt}} \psi' \left(\frac{B_{jt}^{*}}{P_{jt}}\right)} \beta C_{jt+1}^{-\gamma},$$

where $1 + r_{jt} = \left(1 + r_{jt}^{n}\right) \frac{P_{jt}}{P_{jt+1}}$ and $\left(1 + r_{jt}^{*}\right) = \left(1 + r_{t}^{n*}\right) \frac{P_{jt}}{P_{jt+1}}$ are the real interest rates earned on holding the domestic and international bond.

The consumption bundle C_{jt} is a CES aggregate of quantities $q_{ijt}(\omega)$ supplied by firms indexed by ω, from all countries i serving market j:

$$C_{jt} = \left(\sum_{i} \int_{\Omega_{ijt-1}} q_{ijt}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the demand elasticity and Ω_{ijt-1} denotes the endogenous set of varieties produced in country i and available for purchase in country j at time t. Since this set is determined by firms' exporting decisions in the previous period, it is indexed with subscript $t-1$. Demand for each variety ω and the ideal price index satisfy:

$$q_{ijt}(\omega) = C_{jt} \left(\frac{p_{ijt}^{c}(\omega)}{P_{jt}}\right)^{-\sigma}, \tag{2.2}$$

$$P_{jt} = \left(\sum_{i} \int_{\Omega_{ijt-1}} \left(p_{ijt}^{c}(\omega)\right)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}},$$

where $p_{ijt}^{c}(\omega)$ is the price faced by the consumer in country j.

Firms. Firms are monopolistically competitive, face the downward-sloping demand curves given by (2.3), and take the ideal price index as given. The production function is linear in labor. Shipments from country i to j are subject to iceberg transport costs κ_{ijt}, so that

$$q_{ijt}(\omega) = \frac{1}{\kappa_{ijt}} l_{ijt}(\omega), \tag{2.3}$$

where $l_{ijt}(\omega)$ is the firm's labor input for producing for market j. The marginal cost of serving market j is therefore $\kappa_{ijt} w_{it}$. Profit-maximizing firms charge a constant markup over marginal cost:

$$p_{ijt}^{x}(\omega) = \frac{\sigma}{\sigma - 1} \kappa_{ijt} w_{it}, \tag{2.4}$$

where $p_{ijt}^{x}(\omega)$ is the price received by the exporter. As a result, per-period profits are a constant fraction of firm revenue:

$$\pi_{ijt}(\omega) = \frac{1}{\sigma} p^x_{ijt}(\omega)\, q_{ijt}(\omega) = \frac{1}{\sigma} x_{ijt}(\omega). \tag{2.5}$$

Entry. Every period there is a unit mass of potential firms that can enter market j from i. Entry is subject to a stochastic sunk cost of $\xi^s_{ijt}(\omega)$ units of country i's labor, as in Das, Roberts, and Tybout (2007). A firm ω from i that pays the sunk costs in period t sells to j from $t+1$ onwards until it exits. Exit is random and occurs with probability δ. The value of exporting is therefore

$$v_{ijt}(\omega) = \frac{1}{1+r^n_{it}} \left[\pi_{ijt+1}(\omega) + (1-\delta)\, v_{ijt+1}(\omega)\right]. \tag{2.6}$$

A potential entrant enters if the value of exporting exceeds the sunk cost of entry. The marginal firm's sunk costs $\bar{\xi}^s_{ijt}$ satisfy

$$v_{ijt}(\omega) = w_{it} \bar{\xi}^s_{ijt}(\omega). \tag{2.7}$$

Denote by n_{ijt} the mass of exporters from i to j. Its law of motion is

$$n_{ijt} = (1-\delta)\, n_{ijt-1} + G\left(\bar{\xi}^s_{ijt}\right), \tag{2.8}$$

where G denotes the cumulative distribution function of ξ^s_{ijt}.

Tariffs, aggregation, and market clearing. Let τ_{ijt} denote gross *ad valorem* tariffs.[8] Then the prices paid by the consumers and prices received by the exporters satisfy

$$p^c_{ijt}(\omega) = \tau_{ijt} p^x_{ijt}(\omega), \tag{2.9}$$

and the government collects $(\tau_{ijt}-1)p^x_{ijt}(\omega)$ in revenue per unit sold.

Total exports from i to j exclusive of tariff payments are:

$$X_{ijt} = \int_{\Omega_{ijt-1}} x_{ijt}(\omega)\, d\omega = n_{ijt-1} x_{ijt}. \tag{2.10}$$

The tariff revenue of government j is $R^g_{jt} = \sum_i \left(\tau_{ijt}-1\right) X_{ijt}$, and profits in country j are

$$\Pi_{jt} = \sum_i \int_{\Omega_{jit-1}} \pi_{jit}(\omega)\, d\omega - \sum_i \int_{\Omega^e_{jit}} w_{jt} \xi^s_{jit}(\omega)\, d\omega, \tag{2.11}$$

where $\Omega^e_{jit} = \left\{\omega \in [0,1] : \bar{\xi}^s_{jit} \geq \xi^s_{jit}(\omega)\right\}$ is the set of entrants.

[8] In this notation, a 5% *ad valorem* tariff implies $\tau_{ijt} = 1.05$.

Lastly, all countries' labor markets clear:

$$L_i = \sum_j \int_{\Omega_{ijt-1}} l_{ijt}(\omega)\, d\omega + \sum_j \int_{\Omega^e_{ijt}} \xi^s_{ijt}(\omega)\, d\omega + \psi\left(\frac{B^*_{jt}}{P_{jt}}\right), \tag{2.12}$$

and the market for international bonds clears, $\sum_j B^*_{jt} = 0$.

2.2 Steady State Welfare Gains from Trade

In this subsection, we abstract from tariff revenues: $\tau_{ijt} = 1$ for all i and j, implying that $R^g_{jt} = 0$. Since all operating firms in the model have identical quantities and prices, we will drop the firm index ω going forward. Steady state objects are identified by dropping the time subscripts.

Due to the presence of bond holding costs ψ, bond holdings and the associated costs are zero in steady state: $B^*_j/P_j = \psi\left(B^*_j/P_j\right) = 0$. It then follows from the budget constraint (2.1) that real consumption is:

$$C_j = \frac{w_j L_j + \Pi_j}{P_j}. \tag{2.13}$$

We will denote the gross proportional gains from trade as the ratio of real consumption under the current trade regime relative to autarky:

$$GFT = \frac{C_j}{C^{AUT}_j}.$$

In the tradition following Eaton and Kortum (2002) and Arkolakis, Costinot, and Rodríguez-Clare (2012), we seek to express (2.13) as a function of the domestic trade share and exogenous parameters. We start with the standard step that the domestic trade share is:

$$\lambda_{jj} \equiv \frac{n_{jj} x_{jj}}{Y_j} = \frac{n_{jj}\left(\frac{\sigma}{\sigma-1} w_j\right)^{1-\sigma}}{P_j^{1-\sigma}}, \tag{2.14}$$

where $Y_j \equiv P_j C_j$ is total expenditure. Solving this expression for the price index and combining the result with equation (2.13) implies that real consumption satisfies:

$$C_j \propto \frac{w_j L_j + \Pi_j}{w_j \lambda_{jj}^{-\frac{1}{1-\sigma}} n_{jj}^{\frac{1}{1-\sigma}}}. \tag{2.15}$$

From here, we proceed to show that under a specific functional form assumption for the sunk cost distribution $G(\xi^s)$: (i) aggregate profits are a constant fraction of labor income; and that (ii) the mass of domestic firms n_{jj} is a power function of λ_{jj}. The functional form of $G(\xi^s)$ we use is an inverse Pareto distribution:

$$G(\xi^s) = (b\xi^s)^\chi, \tag{2.16}$$

where $\chi > 0$ is the Pareto dispersion parameter and $b > 0$ is the location parameter, that defines the domain of this distribution: $0 < \xi^s \leq \frac{1}{b}$. We assume throughout that b is sufficiently small to ensure that not all potential entrants find it worthwhile to enter in any given period ($\bar{\xi}^s_{ijt} < \frac{1}{b}$ for all t). Under this assumption, the steady state mass of firms becomes

$$n_{ji} = \frac{1}{\delta} \left(b \bar{\xi}^s_{ji} \right)^{\chi}. \tag{2.17}$$

Since $1 + r_j = 1/\beta$ in the steady state, the value of selling to i is:

$$v_{ji} = \frac{\beta}{1 - \beta (1 - \delta)} \pi_{ji} = \frac{\beta}{1 - \beta (1 - \delta)} \frac{1}{\sigma} x_{ji},$$

and the threshold sunk cost of entry is:

$$\bar{\xi}^s_{ji} = \frac{\beta}{1 - \beta (1 - \delta)} \frac{1}{\sigma} \frac{x_{ji}}{w_j}. \tag{2.18}$$

Equations (2.17) and (2.18) imply that the mass of firms is a power function of per-unit sales normalized by the source country wage:

$$n_{ji} \propto \left(\frac{x_{ji}}{w_j} \right)^{\chi}. \tag{2.19}$$

Combining (2.11), (2.12), (2.17), and (2.18), while noting that both bond holdings and bond holding costs are zero in steady state, leads to the desired result that total profits are a constant multiple of labor income:

$$\Pi_j = \frac{\frac{1}{\sigma} \left(1 - \frac{\chi}{\chi+1} \frac{\beta}{1-\beta(1-\delta)} \delta \right)}{1 - \frac{1}{\sigma} \left(1 - \frac{\chi}{\chi+1} \frac{\beta}{1-\beta(1-\delta)} \delta \right)} w_j L_j. \tag{2.20}$$

Finally, starting with the expression for steady state n_{jj} in (2.19), and utilizing (2.13), (2.14), and (2.20) leads to:

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}}. \tag{2.21}$$

Combining (2.15) with (2.20) and (2.21) then yields the result that real consumption is proportional to the domestic trade share:

$$C_j \propto \lambda_{jj}^{\frac{1}{1-\sigma}} n_{jj}^{-\frac{1}{1-\sigma}} = \lambda_{jj}^{\frac{1}{(1-\sigma)(1+\chi)}}. \tag{2.22}$$

Since in autarky $\lambda_{jj} = 1$, (2.22) is also the gains from trade.

Note the difference with the ACR formula for the static Krugman model, $\lambda_{jj}^{\frac{1}{1-\sigma}}$, which would obtain in a setting in which n_{jj} is either exogenously fixed or constant across equilibria. Compared to the classic case and holding σ fixed across models, the gains from trade are moderated because international trade leads to the reduction in domestic varieties. The log change in real consumption following a change in the domestic trade share can be written as:

$$
\begin{aligned}
d\ln C_j &= \frac{1}{1-\sigma} d\ln\lambda_{jj} - \frac{1}{1-\sigma}\frac{d\ln n_{jj}}{d\ln\lambda_{jj}} d\ln\lambda_{jj} \\
&= \underbrace{\frac{1}{1-\sigma} d\ln\lambda_{jj}}_{\text{Gain from foreign varieties}} \quad \underbrace{-\frac{1}{1-\sigma}\frac{\chi}{1+\chi} d\ln\lambda_{jj}}_{\text{Loss of domestic varieties}}.
\end{aligned} \tag{2.23}
$$

The first term is the usual direct effect of the change in the interior trade share, interpreted as the consumption gains from the availability of foreign goods. It increases with trade openness (recall that an increase in trade openness is a fall in λ_{jj}). The second term is the welfare reduction from the loss of domestic varieties, as an increase in trade openness unambiguously lowers n_{jj}. It contributes negatively to the gains from trade. In this model, however, the net gain from openness is positive.

Two further points are worth noting. First, the loss of domestic varieties was modeled and quantified by Melitz and Redding (2015) and Feenstra and Weinstein (2017) in specific static models. We build on these contributions by deriving a parsimonious functional form (2.21) that relates domestic variety to the domestic trade share, which in turn leads to the closed-form GFT expression (2.22) requiring only data on λ_{jj}. As we show below, this property extends to several alternative microfoundations, implying these models admit the same analytical GFT formula.[9] Second, (2.23) together with (2.19) highlight the role of the Pareto dispersion parameter χ. As evident from (2.19), χ is the elasticity of the mass of domestic varieties to the domestic profit opportunities $\frac{x_{jj}}{w_j}$ in steady state. When χ is higher, domestic variety is more sensitive to the profit opportunities, and so the fall in $\frac{x_{jj}}{w_j}$ due to import competition leads to a larger decline in domestic variety, and hence a larger second term in (2.23). When χ is low, the opposite is true: import competition does not move domestic variety much, and thus the second term in (2.23) is smaller in magnitude.[10]

The long-run trade elasticity. A key reason for ACR result's appeal is that the exponent on the domestic trade share is the inverse of the trade elasticity. We now show that the dynamic GFT formula shares this feature. Recall that bilateral trade flows are given by (2.10). Further, in steady state we can write $X_{ij}\left(\kappa_{ij},\tau_{ij}\right) = n_{ij}\left(x_{ij}\left(\kappa_{ij},\tau_{ij}\right)\right) \cdot x_{ij}\left(\kappa_{ij},\tau_{ij}\right)$. The long-run trade elasticity with

[9] In the Krugman model in this section, the term "(mass of) domestic variety" is unambiguous, and refers to n_{jj}. Melitz and Redding (2015) work with a heterogeneous firm model, in which the mass of *potential* varieties (governed by the free entry condition) is distinct from the mass of actually-produced varieties (governed by the zero profit condition). In their analysis, the masses of both potential and actually-produced varieties adjust following trade cost changes, but there is no closed-form characterization of the GFT as in (2.22) (and the model is static). As will become clear in Section 3.1, in our dynamic Melitz model, the n_{jj} margin will correspond to the changes in the mass of *potential* varieties, but both potential and actually-produced varieties will of course adjust following trade cost shocks. Feenstra and Weinstein (2017) do not fully specify the supply side, so it is not pinned down what share of the variety adjustment is due to movements in the free entry vs. zero cutoff profit conditions. Instead, they measure domestic variety adjustments directly with data.

[10] Based on the findings of Dhingra and Morrow (2019) and Bilbiie, Ghironi, and Melitz (2019), one may suspect that the equilibrium in our class of dynamic models is efficient. We have verified that the closed-economy equilibrium of the Krugman model in this section is efficient.

respect to iceberg trade costs therefore has the following components:

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln n_{ij}}{\partial \ln x_{ij}}\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} + \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}}.$$

It is immediate that $\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} = 1-\sigma$, as usual. From (2.19), $\frac{\partial \ln n_{ij}}{\partial \ln x_{ij}}\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} = \chi(1-\sigma)$. Together, the long-run trade elasticity is

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = (1-\sigma)(1+\chi),$$

and thus the gains from trade formula (2.22) features the inverse of the trade elasticity. Note that as in ACR and everywhere else in the literature, this is a partial elasticity that holds constant general-equilibrium changes in expenditures, wages, and prices.

3. General Result

We now state the set of conditions under which the dynamic GFT formula holds.

Proposition 3.1. *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

A.1 For all countries j, trade is balanced (expenditure = revenue):

$$Y_j = w_j L_j + \Pi_j,$$

where $Y_j = C_j P_j$.

A.2 For all countries j, profits are a constant share of GDP:

$$\frac{\Pi_j}{Y_j} = const$$

A.3 For all country pairs (i,j) trade flows satisfy

$$X_{ij} = n_{ij} x_{ij} \tag{3.1}$$

where

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi} \tag{3.2}$$

for some constant $\chi > 0$ and domestic per-unit-mass sales satisfy

$$x_{jj} \propto Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{3.3}$$

where $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij} / \partial \ln \kappa_{ij} < 0$ is the elasticity of exports per unit mass with respect to iceberg trade costs.

Then

$$C_j \propto \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0 (1+\chi)}} \tag{3.4}$$

where $\lambda_{jj} = \frac{X_{jj}}{Y_j}$, and $\varepsilon_\kappa^0(1+\chi)$ is the long-run elasticity of trade flows with respect to iceberg trade costs.

Proof. See Appendix A. □

Note that since $\lambda_{jj} = 1$ in autarky, (3.4) is also the gross proportional gains from trade in steady state. Assumptions A.1 and A.2 are identical to R1 and R2 in ACR. Assumption A.3 puts structure on supply and demand. Condition (3.1) stipulates that total exports from i to j can be written as a product of some generic mass n_{ij} and sales per unit mass x_{ij}. This is without loss of generality, as we can always express total exports as some (average) sales per firm/variety/HS code/etc. times the total number/mass of those units. Condition (3.3) states that domestic demand per unit mass is CES. It parallels ACR's R3. Note that the proposition is stated in terms of the functional form for domestic sales. By doing so the proposition covers a greater range of models. In some models, such as Krugman and customer capital, international trade flows x_{ij} take the same form, modulo iceberg costs κ_{ij}. In the Melitz model, domestic sales per unit mass satisfy (3.3), while export sales contain additional terms, as will become clear below. Closely related, the proposition above states as part of Assumption A.3 that ε_κ^0 is also the elasticity of export sales per unit mass to iceberg costs. Sections 2 and 3.1 make it clear that this condition holds in all the models covered by the proposition that we consider.

Finally, condition (3.2) is a restriction on the functional form for the mass n_{ij}. Qualitatively, it is intuitive: entry increases in the ratio of sales to unit costs. Section 2, in particular equations (2.5) and (2.7), illustrate how this can arise: the value of exporting scales with per period profits, which are in turn proportional to sales (numerator). Sunk costs are paid in terms of domestic labor (denominator). However, the proposition requires more than an increasing relationship: it requires that entry is a power function of this ratio. Section 2 shows how the inverse Pareto distribution of sunk costs leads to (3.2) in the Krugman case. Condition (3.2) differs from ACR, whose result obtains when n_{ij} is constant.

3.1 Mapping from specific models

Section 2 shows that the dynamic Krugman model satisfies the conditions of Proposition 3.1, with $\varepsilon_\kappa^0 = 1 - \sigma$. We now discuss two additional commonly used dynamic models: the customer base model and the Melitz-Pareto model.

Customer base model. In the customer base model (e.g. Arkolakis, 2010; Drozd and Nosal, 2012; Gourio and Rudanko, 2014; Fitzgerald, Haller, and Yedid-Levi, 2024), firms gradually build up the mass of customers they serve. Let there be a country i representative firm that faces downward-sloping demand (2.3) per unit mass of customers in country j. As above, its profits per unit mass of customers are given by (2.5). Let n_{ijt} be the mass of customers that the firm serves. This mass depreciates at rate δ and can be built up by customer acquisition (or investment) a_{ijt}. Thus, the customer mass evolves according to:

$$n_{ijt} = (1-\delta)\, n_{ijt-1} + a_{ijt}. \tag{3.5}$$

Investment of a_{ijt} requires $f(a_{ijt})$ units of labor in the source country. The firm chooses the path of customer base investment to maximize the present value of profits:

$$\max_{\{a_{ijt+s}\}} \sum_{s=0}^{\infty} m_{it,t+s}^{n} \left[n_{ijt+s-1}\pi_{ijt+s} - w_{it} f\left(a_{ijt+s}\right)\right] \tag{3.6}$$

subject to (3.5), where $m_{it,t+s}^{n}$ is the firm's discount factor. The first-order conditions of this problem can be manipulated to yield:

$$w_{it} f'\left(a_{ijt}\right) = v_{ijt}, \tag{3.7}$$

$$v_{ijt} = \frac{1}{1+r_{it}^{n}} \left(\pi_{ijt+1} + (1-\delta)\, v_{ijt+1}\right), \tag{3.8}$$

where we assumed that the discount factor of the firm coincides with that of the representative consumer. Let the cost of accessing customers be given by:

$$f\left(a_{ijt}\right) = \frac{\chi}{(1+\chi)\,\zeta} \left(a_{ijt}\right)^{\frac{1}{\chi}+1}. \tag{3.9}$$

Then, in steady state:

$$n_{ij} = \frac{1}{\delta} a_{ij} = \frac{1}{\delta}\left(\zeta \frac{v_{ij}}{w_i}\right)^{\chi}. \tag{3.10}$$

In turn, combining (2.5) and (3.8) yields the proportionality of v_{ij} to x_{ij}, verifying assumption A.3 in Proposition 3.1.

To see that Assumption A.2 is satisfied, note that aggregate profits can be written as:

$$\Pi_i = \sum_j \left(\frac{1}{\sigma} n_{ij} x_{ij} - w_i \frac{\chi}{(1+\chi)\,\zeta}\left(a_{ij}\right)^{\frac{1}{\chi}+1}\right). \tag{3.11}$$

Since a_{ij} is proportional to n_{ij}, and $a_{ij}^{\frac{1}{\chi}}$ is proportional to x_{ij}/w_i (see 3.10 for both), $\left(a_{ijt}\right)^{\frac{1}{\chi}+1}$ is proportional to $n_{ij}x_{ij}$, and w_i cancels out in the consumer base cost term.

The deeper microfoundation, and thus the interpretation of some equilibrium quantities (e.g., n_{ij}) or parameters (e.g., χ) are different from the Krugman model. However, this model is isomorphic to the dynamic Krugman model in its predictions for aggregate trade flows and the functional forms of the trade elasticities.

Melitz-Pareto. The dynamic Melitz (2003) model differs from the Krugman model in Section 2 in two ways. First, firms are heterogeneous in productivity, denoted $\varphi(\omega)$. Continuing to assume constant Dixit-Stiglitz markups, a firm ω's price becomes:

$$p_{ijt}^{x}(\omega) = \frac{\sigma}{\sigma-1}\frac{\kappa_{ijt}w_{it}}{\varphi(\omega)}. \tag{3.12}$$

We assume that $\varphi(\omega)$ is distributed Pareto:

$$F(\varphi) = 1 - \left(\frac{\varphi_L}{\varphi}\right)^{\theta}. \tag{3.13}$$

Second, the firm in i needs to pay a per-period fixed cost ξ_{ij} denominated in units of i's labor in order to serve market j.

As in Section 2, each period begins with a unit mass of potential entrants from each market i to each market j. Potential entrants do not know their productivity. To put themselves in the position to serve market j, a potential entrant must pay a stochastic sunk cost $\xi_{ijt}^{s}(\omega)$ that (i) allows them to sell to market j, and (ii) reveals its productivity for serving market j. Thus, the decision whether or not to pay the sunk cost is made based on expected discounted profits. As above, the sunk cost is drawn from an inverse Pareto distribution (2.16).[11]

Due to the per-period fixed cost not all firms n_{ijt-1} that have paid the sunk cost will end up exporting. The marginal actually-producing firm earns variable profits that just cover the per-period fixed costs: $\frac{1}{\sigma}x_{ijt}(\omega) = w_{it}\xi_{ij}$. Combining (2.3) and (3.12) (and noting that without tariffs $p_{ijt}^{x}(\omega) = p_{ijt}^{c}(\omega)$) leads to the productivity cutoff for selling from i to j:

$$\varphi_{ijt}^{m} = \frac{\sigma}{\sigma-1}\kappa_{ijt}w_{it}\left(\frac{\sigma w_{it}\xi_{ij}}{C_{jt}\left(P_{jt}\right)^{\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{3.14}$$

Total sales from i to j are:

$$\begin{aligned} X_{ijt} &= \int x_{ijt}(\omega)\,d\omega = n_{ijt-1}\int_{\varphi_{ijt}^{m}}^{\infty} x_{ijt}(\varphi)\,dF(\varphi) \\ &= n_{ijt-1}\underbrace{C_{jt}\left(P_{jt}\right)^{\sigma}\left(\left(\frac{\theta\varphi_L^{\theta}}{\theta-(\sigma-1)}\right)^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\frac{\kappa_{ijt}w_{it}}{\left(\varphi_{ijt}^{m}\right)^{\frac{\sigma-1-\theta}{\sigma-1}}}\right)^{1-\sigma}}_{=:x_{ijt}}, \end{aligned} \tag{3.15}$$

[11] Unlike in the standard Melitz model, the productivity of firms is destination specific in our environment. This assumption allows us to maintain tractability.

where the last line comes from applying the Pareto distribution. Relative to the Krugman model, the sales per unit mass of potential firms, x_{ijt}, are affected by entry/exit of the marginal firms – movements in φ^m_{ijt}.[12] Combining (3.14) and (3.15) leads to the following expression for φ^m_{ijt}:

$$\varphi^m_{ijt} = \left(\frac{\theta\varphi_L^\theta \xi_{ij}\sigma}{\theta-(\sigma-1)}\frac{w_{it}}{x_{ijt}}\right)^{\frac{1}{\theta}}. \tag{3.16}$$

In turn, combining (3.15) and (3.16) produces the following expression for x_{ijt}:

$$x_{ijt} \propto \left(\frac{Y_{jt}}{w_{it}}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}} Y_{jt}\left(\frac{\kappa_{ijt}w_{it}}{P_{jt}}\right)^{-\theta}. \tag{3.17}$$

Equation (3.17) clarifies that in the Melitz model, sales per unit mass involve an additional term $\left(Y_{jt}/w_{it}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}}$ that is absent from Krugman and customer capital models. This term arises due to the extensive margin, whereby the cutoff for serving a market is a function of market size Y_{jt}, scaled by the domestic unit costs: if market size increases, more firms will enter and hence increase sales per unit mass.

Thus, unlike in the Krugman model, in the Melitz model there are 2 distinct notions of changing variety: (i) due to adjustments in the mass of entrants n_{ij}, and (ii) due to the movements in the production cutoff φ^m_{ijt}. The notion of the mass of potential, or "innovated" firms n_{ij} in equation (3.2) in Proposition 3.1 refers exclusively to (i), and is what distinguishes our result from ACR. The original ACR result applies when (i) is fixed, but allows for (ii) – as do we.

Even though foreign sales do not follow a simple CES demand functional form, domestic sales do. If A.2 holds, then the ratio Y_{jt}/w_{jt} is constant and x_{jj} conforms to (3.3) in Proposition 3.1. We show below that A.2 holds.

In steady state, at the time sunk costs are paid, expected profits are:

$$E\left[\pi_{ij}(\omega)\right] = \frac{1}{\sigma}x_{ij} - w_i\xi_{ij}\left(\frac{\varphi_L}{\varphi^m_{ij}}\right)^{\theta}. \tag{3.18}$$

Combining with (3.16) leads to the familiar result that expected profits are a constant fraction of expected sales: $E\left[\pi_{ij}(\omega)\right] = \frac{\sigma-1}{\theta}\frac{x_{ij}}{\sigma}$. Since (2.17) and (2.18) hold unchanged in the Melitz model (with the qualification that here, x_{ij} is sales per unit mass of firms rather than representative firm sales), they lead to (3.2), and Assumption A.3 is satisfied.

[12]Note that sales per unit mass of potential sellers, $x_{ijt} = \int_{\varphi^m_{ijt}}^{\infty} x_{ijt}(\varphi)\, dF(\varphi)$, is not the same as the average sales of firms serving a market, which is $x_{ijt}/(1-F(\varphi^m_{ijt}))$. When market size increases, φ^m_{ijt} falls – less productive firms enter. This increases x_{ijt} since a higher fraction of firms per unit mass sell to the market. At the same time, the average productivity of sellers falls, as less productive firms can serve larger markets.

To see that A.2 is satisfied, note that the steady state profits of country i firms from selling to j are:

$$\begin{aligned}\Pi_{ij} =& \frac{\sigma-1}{\theta}\frac{1}{\sigma}n_{ij}x_{ij} - w_i\int_0^{\bar{\xi}^s{}_{ij}} \xi^s g\left(\xi^s\right) d\xi^s \\ =& \frac{\sigma-1}{\theta}\frac{1}{\sigma}n_{ij}x_{ij} - \frac{\chi}{\chi+1}\beta\frac{\sigma-1}{\theta\sigma}x_{ij}n_{ij} \\ =& \left(1-\frac{\chi}{\chi+1}\frac{\beta\delta}{1-\beta\left(1-\delta\right)}\right)\frac{\sigma-1}{\theta\sigma}X_{ij},\end{aligned}$$

where the second line uses the distributional assumption on the sunk costs ξ^s. Summing across destinations and imposing trade balance delivers Assumption A.2.

We obtain the familiar result that the elasticity of x_{ij} with respect to trade costs κ_{ij}, ε_κ^0, is no longer a function of the elasticity of substitution, but of the dispersion parameter in the Pareto productivity distribution. Relative to the Krugman model, following a change in trade costs, average sales per unit mass x_{ijt} will change both because of the intensive margin (all firms' sales change) and the extensive margin (marginal firms entering/exiting). As in Chaney (2008), Arkolakis et al. (2008), and ACR, when it comes to x_{ij}, those two margins' net effect is captured by $-\theta$.

Differently from those static models, and as in the Krugman model in Section 2, the gains from trade are conditioned not just by θ, but also by the parameter of the sunk cost distribution χ, due to the adjustment of the mass of firms n_{jj} that pay the sunk costs to obtain productivity draws. Thus, the Melitz extension retains the intuitions laid out in Section 2.

3.2 Generalization to Tariffs

Often, trade elasticities are estimated using variation in tariffs. To build up towards measurement and quantification, we state a generalization of Proposition 3.1 to a case with tariffs.

Proposition 3.2. *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

A.1′ For all countries j, trade is balanced (expenditure = revenue):

$$Y_j = w_j L_j + \Pi_j + R_j^g,$$

where $Y_j = C_j P_j$ and $R_j^g = \sum_i \left(\tau_{ij}-1\right) X_{ij}$.

A.2′ For all countries j, profits are a constant share of labor income:

$$\frac{\Pi_j}{w_j L_j} = const$$

A.3′ For all country pairs (i,j) trade flows satisfy

$$X_{ij} = n_{ij} x_{ij} \tag{3.19}$$

where

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi} \tag{3.20}$$

for some constant $\chi > 0$ and domestic per-unit-mass sales satisfy

$$x_{jj} \propto Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{3.21}$$

where $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij} / \partial \ln \kappa_{ij} < 0$ is the elasticity of exports per unit mass with respect to iceberg trade costs.

Then

$$C_j \propto \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0 (1+\chi)}} \left(1 - \frac{R_j^g}{Y_j}\right)^{-\left(1 - \frac{\chi}{1+\chi}\frac{1}{\varepsilon_\kappa^0}\right)}, \tag{3.22}$$

and $\varepsilon_\kappa^0(1+\chi)$ is the long-run elasticity of trade flows with respect to iceberg trade costs.

Proof. See Appendix A. □

Note that $\lambda_{ij} \equiv \frac{\tau_{ij} X_{ij}}{Y_j}$ is now the tariff-inclusive expenditure shares on goods from i. Since $\lambda_{jj} = 1$ and $R_j^g = 0$ in autarky, (3.22) is also the gross proportional gains from trade in steady state.

Because tariffs generate revenue, (3.22) differs from (3.4) by the multiplicative factor that is a function of one minus the tariff revenue share in final expenditure. This multiplicative factor is greater than 1 as long as tariff revenue is positive. Thus, it amplifies the gains from trade relative to the no-tariff formula, conditional on the same λ_{jj}. In static models, this type of tariff adjustment to the ACR formula was to our knowledge first stated by Felbermayr, Jung, and Larch (2015). We show that it operates in a similar way in a dynamic setting. As in Felbermayr, Jung, and Larch (2015), the exponent on the tariff adjustment term cannot be recovered from the long-run trade elasticity alone. We show below how to recover this exponent from estimates of short- and long-run trade elasticities.

The data requirements for computing (3.22) are low. In addition to the domestic trade share, all it additionally requires is the total tariff revenue as a share of GDP. This information is often available from statistical authorities. For the quantification below, we will require bilateral *ad valorem* tariff rates. Thus, it will be convenient to state the following alternative functional form for this adjustment factor:

$$1 - \frac{R_j^g}{Y_j} = \sum_i \frac{1}{\tau_{ij}} \lambda_{ij}.$$

We note that the Melitz-Pareto model with tariffs is not covered by Proposition 3.2, because tariffs also affect the extensive margin conditional on drawing the sunk cost, in a way that is not captured by (3.21). Proposition A.1 in Appendix A is an extension of Proposition 3.2 that also covers the Melitz-Pareto model with tariffs. The extended proposition is identical to Proposition 3.2 except for a strictly more general functional form for sales per unit mass x_{jj}. This generalization only affects the exponent on the tariff adjustment term $\left(1 - R_j^g / Y_j\right)$, and leaves the component of the GFT related to λ_{jj} unaffected. As we show in the quantification below, the tariff adjustment term is not quantitatively important.

In addition, the change to the exponent introduced by the extensive margin in the Melitz-Pareto model vanishes as $\frac{\theta}{\sigma-1} \to 1$ (the "Zipf limit"). In the Melitz-Pareto model $-\frac{\theta}{\sigma-1}$ is the slope of the power law in firm sales to a particular destination. In the data, firm sales follow a power law with an exponent close to -1 (Axtell, 2001; di Giovanni, Levchenko, and Rancière, 2011; di Giovanni and Levchenko, 2013), making the "Zipf limit" an appropriate calibration. Appendix A contains the detailed discussion.

3.3 Ex-ante analysis

The analysis thus far has considered how much welfare changes for a given change in openness as measured by λ_{jj}. We next consider an ex-ante analysis, asking how expenditure shares change for a given change in trade costs. For any variable x, denote by $\widehat{x} \equiv x'/x$ the gross proportional change between the pre-shock steady state value x and the post-shock steady state value x'.

Proposition 3.3. *Consider a class of dynamic models that satisfy the Assumptions A.1'-A.3' in Proposition 3.2, and in addition the trade shares satisfy:*

$$\lambda_{ij} = \frac{n_{ij} \left(\tau_{ij} \kappa_{ij} w_i\right)^{\varepsilon_\kappa^0}}{\sum_k n_{kj} \left(\tau_{kj} \kappa_{kj} w_k\right)^{\varepsilon_\kappa^0}} \tag{3.23}$$

for all i, j.

Then, following changes in iceberg trade costs $\hat{\kappa}_{ij}$ and tariffs $\hat{\tau}_{ij}$ for all $i \neq j$, the steady state change in expenditure shares is given by

$$\hat{\lambda}_{ij} = \frac{\left(\hat{\kappa}_{ij}\right)^{(1+\chi)\varepsilon_\kappa^0} \left(\hat{\tau}_{ij} \hat{w}_i\right)^{\chi\left(\varepsilon_\kappa^0 - 1\right) + \varepsilon_\kappa^0}}{\sum_k \lambda_{kj} \left(\hat{\kappa}_{kj}\right)^{(1+\chi)\varepsilon_\kappa^0} \left(\hat{\tau}_{kj} \hat{w}_k\right)^{\chi\left(\varepsilon_\kappa^0 - 1\right) + \varepsilon_\kappa^0}} \tag{3.24}$$

for all i and j, where the wage changes are determined by the system

$$\hat{w}_i = \sum_j \frac{\frac{1}{\tau_{ij}} \lambda_{ij} \left(\hat{\kappa}_{ij}\right)^{(1+\chi)\varepsilon_\kappa^0} \left(\hat{\tau}_{ij}\right)^{(\chi+1)\left(\varepsilon_\kappa^0 - 1\right)} \left(\hat{w}_i\right)^{\chi\left(\varepsilon_\kappa^0 - 1\right) + \varepsilon_\kappa^0} \hat{w}_j \frac{w_j L_j}{w_i L_i}}{\sum_k \frac{1}{\tau_{kj}} \lambda_{kj} \left(\hat{\kappa}_{kj}\right)^{(1+\chi)\varepsilon_\kappa^0} \left(\hat{\tau}_{kj}\right)^{(\chi+1)\left(\varepsilon_\kappa^0 - 1\right)} \left(\hat{w}_k\right)^{\chi\left(\varepsilon_\kappa^0 - 1\right) + \varepsilon_\kappa^0}} \tag{3.25}$$

for all i.

Proof. See Appendix A. □

Proposition 3.3 is analogous to the ex ante analysis in ACR, but once again with the important difference that in our framework, the masses of firms n_{ji} will adjust following the trade cost shock. Because of the adjustment in n_{ji}, and in contrast to Proposition 3.1, ex ante analysis requires knowledge of ε_{κ}^{0} and χ separately, even if the long-run iceberg trade cost elasticity $\varepsilon_{\kappa}^{0}(1+\chi)$ is known.

Note also that while the Krugman and the customer capital model laid out in Section 3.1 satisfy the restriction (3.23) needed for the proposition, for the Melitz model it depends on whether the per-period fixed costs are paid in the source or destination country's labor. When those costs are paid in the destination country's labor, the trade shares comply with (3.23) up to a vector of multiplicative constants irrelevant for the proof, and thus Proposition 3.3 applies. When the fixed costs are paid in the source country labor, as is more commonly assumed, the trade shares are instead:

$$\lambda_{ij} = \frac{n_{ij}\left(\tau_{ij}\kappa_{ij}w_i\right)^{-\theta}\left(w_i\xi_{ij}\right)^{-\frac{\theta-(\sigma-1)}{\sigma-1}}}{\sum_k n_{kj}\left(\tau_{kj}\kappa_{kj}w_k\right)^{-\theta}\left(w_k\xi_{kj}\right)^{-\frac{\theta-(\sigma-1)}{\sigma-1}}}.$$

While these shares do not comply with (3.23), they can still be used to perform an ex-ante analysis in a way similar to Proposition 3.3, but would require knowledge of one more parameter, σ. In addition, these Melitz trade shares converge to the trade shares (3.23) as $\frac{\theta}{\sigma-1} \to 1$. As noted above and elaborated in the Appendix, $\frac{\theta}{\sigma-1} \approx 1$ (the "Zipf limit") is a calibration favored by the data, and thus (3.23) a good approximation of the trade shares in the Melitz model.

3.4 Endogenous Capital Accumulation

Adding physical capital accumulation to the class of models considered above is not common, as the masses of firms n_{ijt} are themselves a type of capital stock, subject to forward-looking accumulation decisions and depreciation. Nonetheless, to bridge our framework with a wider range of dynamic trade models, we now extend the model to endogenous capital K_{jt}.

Households own capital and invest to accumulate it. The budget constraint and the capital accumulation equation are:

$$P_{jt}\left(C_{jt}+I_{jt}\right)+B_{jt}+B_{jt}^{*}+w_{jt}\psi\left(\frac{B_{jt}^{*}}{P_{jt}}\right)=w_{jt}L_j+r_{jt}^{k}K_{jt}+\Pi_{jt}+R_{jt}^{g}+B_{jt-1}\left(1+r_{jt-1}^{n}\right)+B_{jt-1}^{*}\left(1+r_{t-1}^{n*}\right)$$

and

$$K_{jt+1}=(1-\delta)K_{jt}+I_{jt},$$

where I_{jt} is investment and the rental price of capital is r^k_{jt}. The Euler equation for capital is:

$$\left(\frac{C_{jt+1}}{C_{jt}}\right)^{\gamma} = \beta\left(\frac{r^k_{jt+1}}{P_{jt+1}} + 1 - \delta\right).$$

Production is Cobb-Douglas in capital and labor. Competitive producers assemble capital-labor bundles according to the following production function:

$$b_{it} = \alpha^{\alpha}(1-\alpha)^{1-\alpha} l_{it}^{\alpha} k_{it}^{1-\alpha},$$

where l_{it} and k_{it} are the labor and capital employed by the bundle producers. As a result, the cost of the input bundle is

$$c_{it} = w_{it}^{\alpha}(r^k_{it})^{1-\alpha}.$$

For expositional clarity we specialize the production side to the Krugman model. In that case, production quantities, prices, and per-period profits are given by (2.3)-(2.5), replacing w_{it} by c_{it} and l_{it} by b_{it}. The value of exporting is still given by (2.6), and the marginal firm's sunk costs $\bar{\xi}^s_{ijt}$ satisfy (2.7), once again with w_{it} replaced by c_{it}.

Under these conditions and in steady state total profits are proportional to total factor income: $\Pi_j \propto w_j L_j + r^k_j K_j$. In addition, cost minimization and factor market clearing imply that

$$\frac{w_{jt} L_j}{r^r_{jt} K_{jt}} = \frac{\alpha}{1-\alpha}.$$

Combining these two properties yields the result that in steady state, capital and investment are proportional to consumption:

$$I_j = \delta K_j \propto C_j.$$

This means that real consumption is proportional to the total real income, which is in turn proportional to the real wage income:

$$C_j \propto \frac{w_j L_j + r^k_j K_j + \Pi_j}{P_j} \propto \frac{w_j L_j}{P_j}.$$

In turn, the price index is (see 2.14):

$$P_j = \frac{\sigma}{\sigma-1} c_j \left(\frac{\lambda_{jj}}{n_{jj}}\right)^{\frac{1}{\sigma-1}}.$$

Putting these together, real consumption is proportional to:

$$C_j \propto \left(\frac{\lambda_{jj}}{n_{jj}}\right)^{\frac{1}{1-\sigma}} K_j^{1-\alpha}.$$

Note that when K_j is fixed, the same steps with respect to n_{jj} yield the GFT formula (2.22)/(3.4) as in Sections 2 and 3. When n_{jj} is fixed, we are in a model with constant variety but endogenous capital accumulation, reminiscent of e.g., Alvarez (2017) and Ravikumar, Santacreu, and Sposi (2019). In that case, we can use the proportionality of steady state capital to consumption to arrive at the following GFT formula:

$$C_j \propto \lambda_{jj}^{\frac{1}{1-\sigma}\frac{1}{\alpha}}.$$

Trade leads to capital accumulation, raising the GFT above the "static" level (as $\alpha < 1$), a channel emphasized by Ravikumar, Santacreu, and Sposi (2019) among others.

Finally, when both n_{jj} and K_j adjust following trade opening, as would be the case in the complete model, the GFT formula is

$$GFT = \lambda_{jj}^{\frac{\frac{1}{1+\chi}\frac{1}{1-\sigma}}{1-(1-\alpha)\left(1-\frac{\chi}{1+\chi}\frac{1}{1-\sigma}\right)}}.$$

Relative to the formula in the labor-only model (2.22)/(3.4), the GFT are amplified by $1-(1-\alpha)\left(1-\frac{\chi}{1+\chi}\frac{1}{1-\sigma}\right)$, which comes from the fact that the physical capital stock K_j reacts to trade opening. Note that it is no longer guaranteed that the GFT are positive. The gains are positive when σ is large enough:

$$\sigma > 1 + \frac{\chi}{1+\chi}\frac{1-\alpha}{\alpha}.$$

Under this parameter restriction, the GFT are strictly larger than in the labor-only model (2.22)/(3.4), as the country accumulates K_j following trade opening.

On the other hand, the combination of accumulable K_j and accumulable n_{jj} leads to negative GFT under some parameter values. This is because the complementarity between K_j and n_{jj} leads to an agglomeration force: a larger capital stock lowers the cost of accumulating n_{jj}, and vice versa. Combining expressions for n_{jj}, x_{jj}, and c_j leads to the following steady state relationship:

$$K_j \propto \left(\frac{n_{jj}^{\frac{1+\chi}{\chi}}}{\lambda_{jj}}\right)^{\frac{1}{1-\alpha}}.$$

Domestic capital accumulation has a positive relationship with n_{jj}, and a negative relationship with λ_{jj}, both of which are intuitive. When the loss of domestic variety following trade opening is significant enough relative to the export opportunities encapsulated by λ_{jj}, trade opening reduces the steady state level of capital, and thereby welfare.

4. Measurement

Similar to ACR, the steady state gains from trade in this class of dynamic models are a function of the domestic absorption share and exogenous parameters. Propositions 3.1-3.2 state that the domestic absorption share is raised to the power of the inverse of the long-run iceberg trade elasticity. In dynamic models, this long-run trade elasticity is a function of different structural parameters than the "trade elasticity" in static models. We now show that this has implications for how this long-run elasticity can be recovered from econometric estimates and for the measurement of the gains from trade.

4.1 Trade Elasticities

The exponent in the gains from trade formula is the inverse of the long-run elasticity of trade with respect to iceberg trade costs κ_{ij}:

$$\varepsilon_\kappa \equiv \frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} + \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} (1 + \chi) = \varepsilon_\kappa^0 (1 + \chi). \tag{4.1}$$

Though a few papers have used shipping cost data to estimate the iceberg trade cost elasticity (e.g., Hummels, 2001; Shapiro, 2016; Adão, Costinot, and Donaldson, 2017), the large majority of existing trade elasticity estimates use tariffs. However, the trade elasticity with respect to tariffs differs from that with respect to iceberg costs. The long-run tariff elasticity is:

$$\varepsilon_\tau \equiv \frac{\partial \ln X_{ij}}{\partial \ln \tau_{ij}} = \frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} \frac{\partial \ln x_{ij}}{\partial \ln \tau_{ij}} + \frac{\partial \ln x_{ij}}{\partial \ln \tau_{ij}} = \frac{\partial \ln x_{ij}}{\partial \ln \tau_{ij}} (1 + \chi) = \varepsilon_\tau^0 (1 + \chi), \tag{4.2}$$

where

$$\varepsilon_\tau^0 \equiv \frac{\partial \ln x_{ij}}{\partial \ln \tau_{ij}}. \tag{4.3}$$

In what follows, in line with the majority of the literature, we presume that the researcher has an estimate of ε_τ in hand, but needs ε_κ as required by the ACR and our formulas. The stumbling block is that $\varepsilon_\tau^0 \neq \varepsilon_\kappa^0$: the short-run elasticity of sales per unit mass with respect to τ_{ij} does not equal the elasticity of sales to κ_{ij}. The reason is that iceberg trade costs and tariffs have distinct effects on sales net of tariff payments x_{ij} (see e.g. 2.5). For example, in the Krugman and customer base models $\varepsilon_\kappa^0 = 1 - \sigma$, whereas $\varepsilon_\tau^0 = -\sigma$ (see equations 2.3, 2.4, and 2.9), implying that

$$\varepsilon_\kappa^0 = \varepsilon_\tau^0 + 1. \tag{4.4}$$

In the Melitz model from Section 3.1, instead of being different by 1, the short-run iceberg and tariff elasticities have the following relationship: $\varepsilon_\kappa^0 = \varepsilon_\tau^0 + \frac{\theta}{\sigma-1}$. Hence, in that case equation (4.4) only holds in the Zipf limit as $\frac{\theta}{\sigma-1} \to 1$, but that is the quantitatively relevant case (di Giovanni and Levchenko, 2013, see also appendix A).[13]

[13] Note that this relationship holds also in static Melitz-Pareto implementations in which n_{ij} does not move, and therefore all static ACR quantifications that are intended to cover Melitz models. Thus, in the Melitz case one needs to be more careful when going between the tariff and the iceberg elasticities. Outside of the Zipf limit, obtaining the iceberg elasticity from the tariff elasticity would additionally require the parameter combination $\frac{\theta}{\sigma-1}$. With that number in place of 1, the

The intuition can be built up as follows. When iceberg trade costs are modeled as part of firms' technologies for serving foreign markets – by far the most common approach in the trade literature, also adopted throughout this paper – iceberg trade costs affect the price received by the exporter p_{ij}^{x} while tariffs do not (see equation 2.4). Specifically, a given increase in the iceberg trade cost translates into a higher price received by the exporter. By contrast, a higher tariff does not raise the price received by the exporter because it goes to the government.

At the same time, the prices faced by consumers include both tariffs and iceberg trade costs (see equations 2.4 and 2.9). As a result, changes in tariffs and changes in iceberg trade costs affect consumer prices and demand in the same way —- and therefore have identical effects on consumer expenditures. However, because the government takes part of consumer expenditures as tariff revenue, changes in tariffs and iceberg trade costs affect firm sales and flow profits differently. In response to an increase in tariffs, the government takes a larger cut. After an increase in iceberg trade costs, by contrast, the firm raises its price, thereby offsetting part of the drop in quantities. That is why ε_{κ}^{0} is less in absolute value than ε_{τ}^{0}.[14]

This difference, of course, also arises in static models. Most (though not all) of the literature that estimates trade elasticities in the context of static models recognizes it. When n_{ij} does not respond to trade costs, as is the case in practice in most static models such as Armington or Krugman, this distinction is fairly innocuous. To account for it, most papers either add 1 to the tariff elasticity to recover the iceberg cost elasticity, or use trade flows inclusive of tariff payments in estimation.

In our dynamic setting, or more generally if the mass of exporters responds to trade costs in the long run, neither of these simple adjustments work, requiring another strategy to recover the iceberg elasticity.[15] Combining equations (4.1), (4.2), and (4.4) yields

$$\varepsilon_{\kappa} = \varepsilon_{\tau} + 1 + \chi. \tag{4.5}$$

4.2 Difference from ACR

Continue with the presumption that the researcher has an estimate of ε_{τ}. Equation (4.5) implies that, conditional on ε_{τ}, the gains from trade are greater in our class of dynamic models than under the ACR

rest of the procedure for going from the tariff elasticities to the value required by the GFT formula is unchanged.

[14] An alternative but for our purposes equivalent way of modeling iceberg trade costs is to assume that the difference between consumer and producer prices includes both tariffs and trade costs, because some products "melt" in transit. In this case, neither tariffs nor iceberg trade costs are part of firms' marginal costs and hence neither affects producer prices. However, changes in iceberg trade costs alter the amount that needs to be produced and shipped. Consider again the two thought experiments of increasing tariffs or increasing iceberg trade costs by an equal amount. Producer prices are not affected in this case while consumer prices rise with equal elasticities. As a result, demand falls by an equal amount in both thought experiments. Since higher iceberg trade costs imply that firms must now produce more per unit that arrives in the destination, the drop in the quantity demanded is partially offset by an increase in the amount that ultimately melts. Production therefore falls by less after a rise in iceberg trade costs. Firm sales, which equal the producer price times the quantity produced, and profits therefore also fall by less in the case of an increase in iceberg trade costs compared to an equal increase in tariffs.

[15] It is immediate from (4.1)-(4.2) that adding 1 to ε_{τ} does not recover ε_{κ}. It is also easy to verify that the long-run tariff elasticity of tariff-inclusive trade flows $\partial \ln\left(\tau_{ij} n_{ij} x_{ij}\right) / \partial \ln \tau_{ij}$ also does not equal ε_{κ}, once again because of changes in n_{ij}.

formula. As highlighted throughout, our formula (sans tariffs) coincides with ACR when expressed in terms of the long-run iceberg elasticity: $\lambda_{jj}^{\frac{1}{\varepsilon_\kappa}}$. However, since most applications employ ε_τ, equation (4.5) is required. In the class of models covered by ACR, the analog of (4.5) is simply $\varepsilon_\kappa = \varepsilon_\tau + 1$. Thus, the GFT in the ACR and our dynamic settings are given by:

$$\begin{array}{cc} \text{Static (ACR)} & \text{Dynamic} \\ \lambda_{jj}^{\frac{1}{\varepsilon_\tau+1}} & \lambda_{jj}^{\frac{1}{\varepsilon_\tau+1+\chi}}. \end{array} \tag{4.6}$$

Importantly, because $\chi > 0$, the GFT implied by our formula are unambiguously greater than those implied by ACR for *any* given value of ε_τ. Comparing these two formulas clarifies that the difference between the GFTs is not due to any potential disagreement among researchers about the value of ε_τ, which is in fact the same in the two formulas. For fixed ε_τ, the magnitude of the difference depends exclusively on χ. The greater χ the greater the difference from the static GFT.

Why is the mapping between the iceberg and the tariff elasticity, i.e., equation (4.5), different in dynamic models? The short answer is that in the class of models covered by our theoretical result, the masses n_{ij} adjust. As highlighted in (4.1)-(4.2), the elasticities of n_{ij} to τ_{ij} and κ_{ij} are composites of two elasticities: $\frac{\partial \ln n_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}}$, and analogously for τ_{ij}. The first term is always simply χ: $\frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} = \chi$. However, as discussed above, $\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} > \frac{\partial \ln x_{ij}}{\partial \ln \tau_{ij}}$: a given log increase in tariffs decreases per-firm sales and therefore profits by more than the same increase in iceberg costs. In turn, changes in flow profits drive the firms' entry decisions and ultimately the steady state mass of firms n_{ij}. Thus, an increase in iceberg costs reduces n_{ij} by less than an equal increase in tariffs. This equilibrium adjustment in n_{ij} is absent in the class of models ACR covers, and ultimately gives rise to the different relationship between the long-run iceberg and tariff elasticity (4.5) than simply adding 1.

4.3 Calibrating χ

Given a long-run tariff elasticity ε_τ, relative to ACR our formula requires one additional object: χ. In the Krugman and Melitz models, χ governs the curvature of the sunk cost distribution (2.16) and captures how the mass of firms from i selling in j, n_{ij}, responds to changes in per-firm sales x_{ij}. By contrast, in the customer base model from Section 3.1 χ parameterizes the curvature of the customer acquisition cost function (3.9). In that model n_{ij}, is the mass of customers in j reached by the representative firm from country i, and x_{ij} is sales per customer. While in all cases $\frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} = \chi$, the interpretations of n_{ij} and x_{ij} are different. Thus, ideally the calibration approach should be valid across microfoundations, and not rely on any direct measurement of n_{ij}.

We therefore propose the following calibration strategy. It is immediate from (4.2) and (4.3) that

$$1 + \chi = \frac{\varepsilon_\tau}{\varepsilon_\tau^0}. \tag{4.7}$$

The intuition is that regardless of the microfoundation and the precise interpretation of n_{ij}, χ can be inferred from the difference between the short- and the long-run adjustment to tariff shocks.

Our baseline quantification sources ε_τ and ε_τ^0 from Boehm, Levchenko, and Pandalai-Nayar (2023). That paper shows that all three microfoundations considered above generate a dynamic version of the gravity equation up to a first order approximation (see, in particular, their Section 5, Proposition 3, and Appendix C). Boehm, Levchenko, and Pandalai-Nayar (2023) also show that these models can match the gradual response of trade following tariff changes well (see their Figure 5). Thus, the estimates in that paper are theoretically-consistent sources of the short-run and long-run tariff elasticities.

Boehm, Levchenko, and Pandalai-Nayar (2023) estimate a long-run tariff elasticity of about $\varepsilon_\tau = -2$. In all models laid out above, the "short run" is a time period over which x_{ij} can adjust but n_{ij} cannot, because it is subject to a one period "time-to-build" (see accumulation equation 2.8). In the data, it is an open question what length of time this corresponds to. Since one may be concerned that the trade adjustment for the first few years is driven by mechanisms absent from the class of models we study – such as sticky prices, contractual frictions, certain bilateral quantity adjustment frictions, and the like – we set the short-run tariff elasticity to $\varepsilon_\tau^0 = -1.25$. This is the estimated value of the trade elasticity in year 5 after the tariff shock, and thus is conservative – their estimates are smaller in absolute value for up to five years, and in fact are initially below 1 in absolute value. A short-run elasticity around -1 is also in line with other short-run estimates in the literature. The combination of $\varepsilon_\tau = -2$ and $\varepsilon_\tau^0 = -1.25$ implies $\chi = 0.6$. We will consider larger short-run elasticities (in absolute terms) in robustness exercises below and hence smaller values of χ.[16]

5. Quantification

In this section, we quantify the welfare gains from trade. We make three main points. First, we compute the gains from trade under our preferred estimates of the trade elasticities, taken from Boehm, Levchenko, and Pandalai-Nayar (2023). This exercise shows that the gains from trade are large, and that the quantitative impact of the tariff adjustment to the GFT formula in (3.22) is generally minor. Second, we compare the dynamic GFT to those obtained from the ACR formula. We highlight that GFT can vary widely even conditional on the same long-run tariff elasticity, and that the dynamic formula can yield large GFT even under a high long-run tariff elasticity. Finally, the third part of the section compares the GFT implied by the formula to those computed numerically in a multi-country

[16] We note that there is a subtlety in the Melitz model when defining the short run, because x_{ijt} can adjust along both the intensive margin (same firms selling more, iceberg elasticity $1-\sigma$) and extensive margin (cutoff φ_{ijt}^m changing, iceberg elasticity $(1-\sigma)\,\frac{\theta-(\sigma-1)}{\sigma-1}$). For our purposes, conceptually, the notion of the short-run iceberg elasticity incorporates both margins, resulting in the overall short-run iceberg elasticity in the Melitz model of $1-\sigma+(1-\sigma)\,\frac{\theta-(\sigma-1)}{\sigma-1} = -\theta$. Therefore, mapping our measurement and quantitative results back to the Melitz case entails an implicit assumption that the time period over which the short-run elasticity is measured in practice is long enough for the cutoff φ_{ijt}^m to adjust—reflecting a static decision to change the firm's exporting status without implications for subsequent periods—but not long enough for n_{ijt} to change.

general equilibrium dynamic setting taking into account the transition path.

5.1 Data

The quantification relies on several sources of data. First, computing the gains from trade using (3.4) requires the domestic absorption share λ_{jj}. Typically, domestic absorption is measured from standard datasets such as the OECD Inter-Country Input Output tables (ICIO). The ICIO contains information on all bilateral sectoral expenditures, covering manufacturing and services, and intermediate and final goods. Importantly, it also contains information on expenditure on domestic sectors. However, the ICIO does not contain information on bilateral tariff revenues, so aggregate expenditure and expenditure shares constructed from this source are not tariff-inclusive.

Computing the gains from trade when *ad valorem* tariffs are non-zero (3.22) requires the total tariff revenue as a share of total (tariff-inclusive) spending. Aggregate tariff revenues are available from the World Bank. However, the full quantitative implementation of the dynamic model additionally requires all tariff-inclusive bilateral expenditure shares λ_{ij}. Therefore, aggregate tariff revenues are not sufficient for our purposes.

To construct bilateral tariff revenue, we obtain tariff data from the TRAINS dataset. This database reports the applied tariff by country pair at the Harmonized System (HS) 6-digit level. We link these data to trade flows at the HS-6 level from the BACI version of UN-COMTRADE. To compute tariff revenue, we multiply the bilateral, product-level applied tariffs obtained from TRAINS with bilateral trade flows from BACI:

$$R_{ij}^{g} = \sum_{p} X_{ijp}^{\text{BACI}} \left(\tau_{ijp}^{\text{TRAINS}} - 1 \right),$$

where R_{ij}^{g} is bilateral tariff revenue from goods trade and p is an HS-6 product. BACI does not contain information on services trade flows. Services trade flows are subject to no tariff, so the aggregate bilateral tariff rate imposed by j on i consistent with goods tariff revenues R_{ij}^{g} is:

$$\tau_{ij} - 1 = \frac{R_{ij}^{g}}{X_{ij}^{\text{ICIO}}},$$

where X_{ij}^{ICIO} is total expenditure of j on goods and services from i, sourced from the OECD ICIO database. We can then calculate all tariff-adjusted trade shares λ_{ij}:

$$\lambda_{ij} = \frac{\tau_{ij} X_{ij}^{\text{ICIO}}}{\sum_{k} \tau_{kj} X_{kj}^{\text{ICIO}}}.$$

Our baseline sample includes 67 countries and a rest-of-the-world aggregate in 2006.[17] We validate

[17] Three percent of the observations show positive bilateral goods trade flows in the ICIO but have no tariffs declared in TRAINS. In these cases, we assume there is 0 tariff revenue associated with these pairs.

our tariff revenue measures by comparing $R_j^g = \sum_i R_{ij}^g$ with national tariff revenue obtained from the World Bank. As the World Bank tariff revenue data are provided in local currency, we convert them to US dollars using an annual exchange rate obtained from the same source. Appendix Figure C1 illustrates that our baseline tariff revenue measures are similar to those obtained from the World Bank. The calibration of the full quantitative model in Section 5.3 additionally requires data on real GDP. We obtain these from the Penn World Tables.

5.2 Steady State Gains from Trade

Figure 1 plots the gains from trade for a sample of countries based on (3.22), $\varepsilon_\tau = -2$, and $\varepsilon_\tau^0 = -1.25$ as detailed above. The blue line is simply the formula (3.4) that ignores the tariff adjustment and only uses information on the domestic trade share. The red dots are (3.22), and thus make the tariff adjustment using country-specific tariff revenue data. Conditional on a fixed λ_{jj}, the tariff adjustment unambiguously increases the gains from trade. However, the tariff adjustment is small quantitatively for all but a few countries.

The gains from trade are large. Even the most closed countries – the US, Brazil, China – gain on the order of 25-30% from trade. Jordan's welfare triples, and Malta's quadruples, when they go from autarky to trade.

We now highlight the difference between the GFT implied by the static and dynamic formulas (4.6). Figure 2 displays the percentage point difference between dynamic and static gains as a function of ε_τ and χ, taking the United States ($\lambda_{jj} = 0.91$) as the case study.[18] Our baseline calibration is marked by the black dot. At our preferred values of ε_τ and χ, the dynamic GFT are 17.8 percentage points larger than those implied by the ACR formula (dynamic GFT: 28.3%; ACR formula: 10.5%). The difference in the implied GFT falls to zero as $\chi \longrightarrow 0$. Holding χ constant, the difference also falls as ε_τ increases in absolute value. However, simply having a high ε_τ is not sufficient to ensure a small disparity between dynamic and static GFT. Taking, for example the "textbook" value of $\varepsilon_\tau = -5$ (Costinot and Rodríguez-Clare, 2014), it is still the case that the difference between static and dynamic gains increases as χ increases.

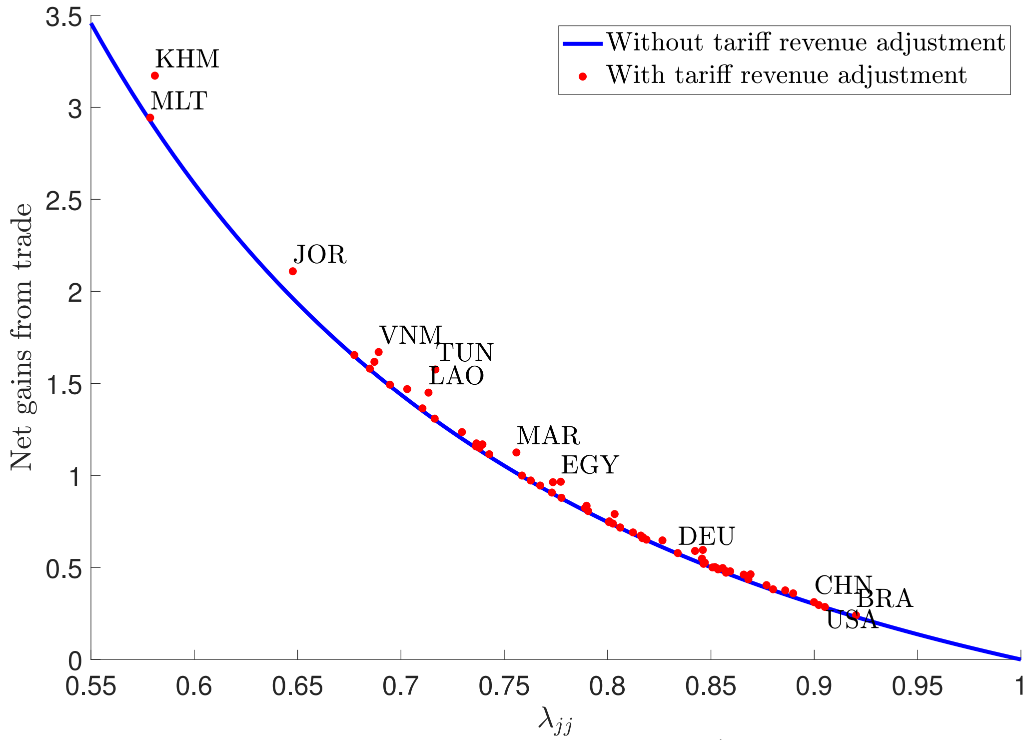
To better understand this result, plug (4.7) into the denominator of the exponent of the dynamic GFT formula to obtain:

$$\varepsilon_\tau + 1 + \chi = \varepsilon_\tau + \frac{\varepsilon_\tau}{\varepsilon_\tau^0}.$$

This expression makes it clear that a high long-run tariff elasticity ε_τ is consistent with arbitrarily large gains from trade if the short-run tariff elasticity is sufficiently low. Indeed, as $\varepsilon_\tau^0 \uparrow -1$, the GFT implied by the dynamic formula become infinite. This is not the case for the static formula, which is completely anchored by ε_τ, and cannot produce large GFT without a low long-run tariff elasticity (a

[18] The heat map is plotted for $\varepsilon_\tau > 1 + \chi$. When this condition is violated the exponent on the dynamic GFT formula turns positive.

Figure 1: Steady State Gains from Trade



Notes: The figure depicts the net proportional welfare gains from trade, $\lambda_{jj}^{\frac{1}{\varepsilon_{\kappa}^{0}(1+\chi)}} - 1$ as a function of the domestic absorption share λ_{jj}. The blue line implements the formula (3.4). The red dots implement the formula that adjusts for tariff revenue, (3.22).

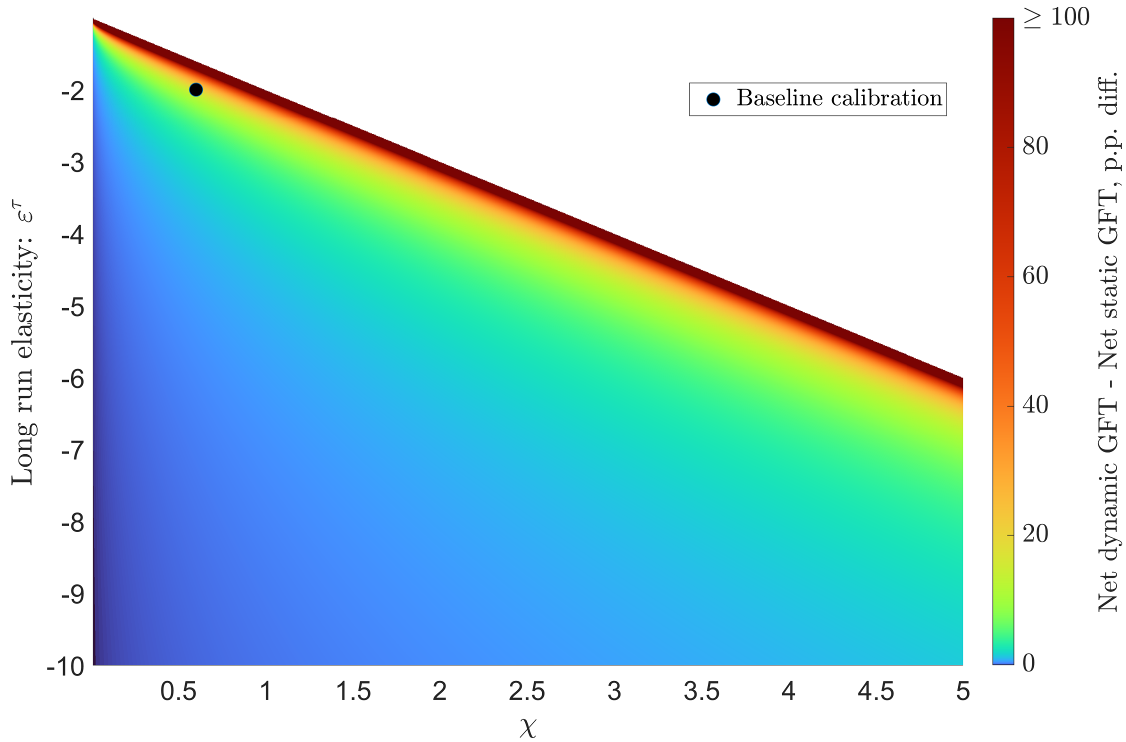
point emphasized by Ossa, 2015). Note that most available estimates of the short-run tariff elasticity are low (Fitzgerald and Haller, 2018; Boehm, Levchenko, and Pandalai-Nayar, 2023; Auer, Burstein, and Lein, 2021), suggesting that the gains from trade are potentially quite large.

5.3 The Transition Path and Welfare Gains

In the final exercise, we answer the question of how costly it is that the formula compares steady states, and thus ignores the transition path. To do this, we compute welfare taking into account the transition between trade regimes. This requires calibrating the full model, and thus taking a stance on all the parameters.

We employ the Krugman model from Section 2. The equilibrium definition, all model equations and the solution approach are provided in Appendix B.1. In addition to σ and χ, calibrated as above using short- and long-run tariff elasticity estimates, we require the depreciation rate δ, risk aversion γ, the discount factor β, the Inverse Pareto scale parameter b, and the adjustment cost function ψ. Of these, the most important one is δ, as it controls the speed of transition. The lower δ is, the slower the transition, and the greater the discrepancy between steady state and fully dynamic gains. When depreciation is full ($\delta = 1$), transition occurs in 1 period. This parameter is disciplined by the speed of convergence of the trade elasticity to the long-run. We set $\delta = 0.25$ to match the

Figure 2: Steady State Gains from Trade: Comparison of Static and Dynamic Formulas



Notes: The figure depicts the percentage point difference in the welfare gains from trade implied by the dynamic GFT formula and the ACR formula in (4.6) for the United States ($\lambda_{jj} = 0.91$) as a function of the tariff elasticity ε_{τ} and χ.

convergence speed estimated in Boehm, Levchenko, and Pandalai-Nayar (2023). As robustness we also consider higher and lower values of δ below. Further, we choose the adjustment cost function $\psi\left(B^{*}_{jt}/P_{jt}\right) = \psi/2\left(B^{*}_{jt}/P_{jt}\right)^{2}$ and calibrate the parameter $\psi = 0.05$ to ensure that the bond holding costs are small. The remaining parameter choices are standard. Table 1 summarizes the baseline calibration. We consider alternative parameter choices for robustness below. We solve the model for the 29 largest countries in the world by GDP and a rest-of-the-world aggregate.

In addition to the observed trade equilibrium our exercises require computation of new steady states that occur if, say, country i moves to autarky. When doing so, we set all pairs of bilateral trade costs κ_{ij} and κ_{ji} for country i to infinity for all $j \neq i$.[19] All other parameters remain unchanged, including the calibrated tariff rates and countries' labor endowments. We then apply Proposition 3.3 to obtain the trade shares λ_{ij} for all country pairs in the new steady state.

While steady state comparisons are unambiguous, in a dynamic setting there are multiple ways in which a country can transition between autarky and trade. First, the direction matters. The consumption stream of transitioning from autarky to trade will differ from the consumption stream associated with transitioning from trade to autarky. Second, assumptions on a country's access to international bond markets will affect its consumption trajectory when opening up to trade. Finally,

[19]In practice, we choose a very large number.

Table 1: Baseline Calibration

Parameters	Value / Target / Source	Notes
σ	1.25	Short-run tariff elasticity
χ	0.6	Inverse Pareto shape parameter
β	0.97	Discount factor
γ	2	Relative risk aversion
δ	0.25	Exit rate
b	1	Inverse Pareto scale parameter
ψ	0.05	Bond holding cost parameter
τ_{ij}	BACI, TRAINS	Average bilateral tariff
κ_{ij}	λ_{ij} from BACI, ICIO, TRAINS	Non-tariff trade costs
L_i	Relative real GDP from PWT	Labor endowment

Notes: The table shows the baseline calibration.

Table 2: Quantitative Exercises

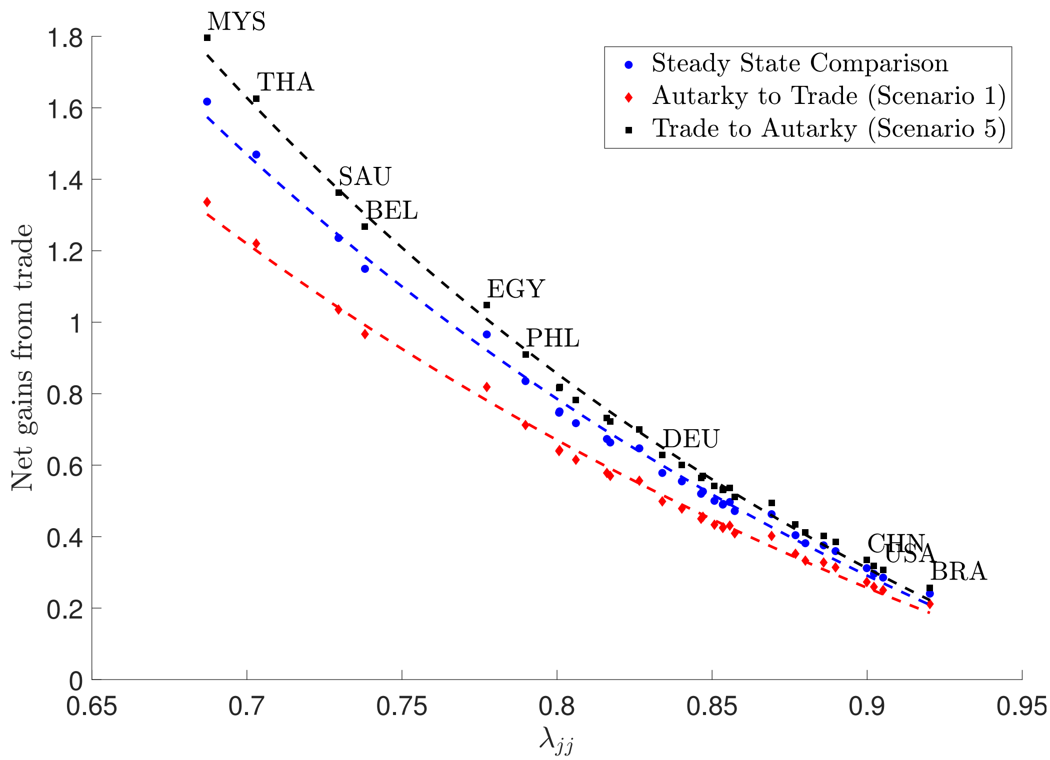
(1) Scenario	(2) Transition	(3) Access to bond markets	(4) Other countries	(5) Net gains from trade (country average)	(6) Cross-sectional correlation with Scenario 1
1 (baseline)	Autarky to Trade	Yes	Open throughout	0.5473	1
2	Autarky to Trade	Yes	Also transitioning	0.5456	1.0000
3	Autarky to Trade	No	Open throughout	0.5451	1.0000
4	Autarky to Trade	No	Also transitioning	0.5451	1.0000
5 (baseline)	Trade to Autarky	*Irrelevant*	*Irrelevant*	0.6979	0.9997

Notes: The table shows the various scenarios considered in the quantitative exercises, the average gains from trade in each scenario, and the cross-sectional correlation of the gains from trade with the baseline scenario 1.

the trade openness of other countries in the world during the transition could impact the consumption stream of an individual country opening to trade. Therefore, we will simulate both the transition from autarky to trade of a single country alone (while all other countries are open throughout) and the transition of all countries from autarky to trade simultaneously. For the transition from trade to autarky, as trade costs are infinite from the first period onwards, both the assumptions on international asset markets and on openness trajectories of other countries are irrelevant. Table 2 provides an overview of these scenarios. In our baseline we consider scenario 1 (the transition of a single country from autarky to trade, assuming that all countries have access to international bond markets) and scenario 5 (the transition from trade to autarky).

Figure 3 reports three sets of gains from trade: (i) comparing autarky and trade steady states according to the formula (3.22) (blue); (ii) transitioning from autarky to the current levels of trade openness (scenario 1 from Table 2, in red); and (iii) transitioning from the current levels of openness to autarky (scenario 5 from Table 2, in black). For (ii) and (iii) we begin in the initial steady state

Figure 3: Steady State Gains vs. Gains over the Transition Path



Notes: The blue dots depict the GFT computed using formula (3.22) and hence represent steady state comparisons. The red dots depict the GFT for a single country starting in autarky and moving to the observed levels of trade, relative to remaining in autarky forever. The country has access to bond markets and all other countries' trade costs remain unchanged in this exercise (Scenario 1). The black dots depict the GFT for a single country trading forever relative to transitioning to autarky (Scenario 5). Dashed lines represent an exponential fit between the gains from trade and the domestic absorption shares.

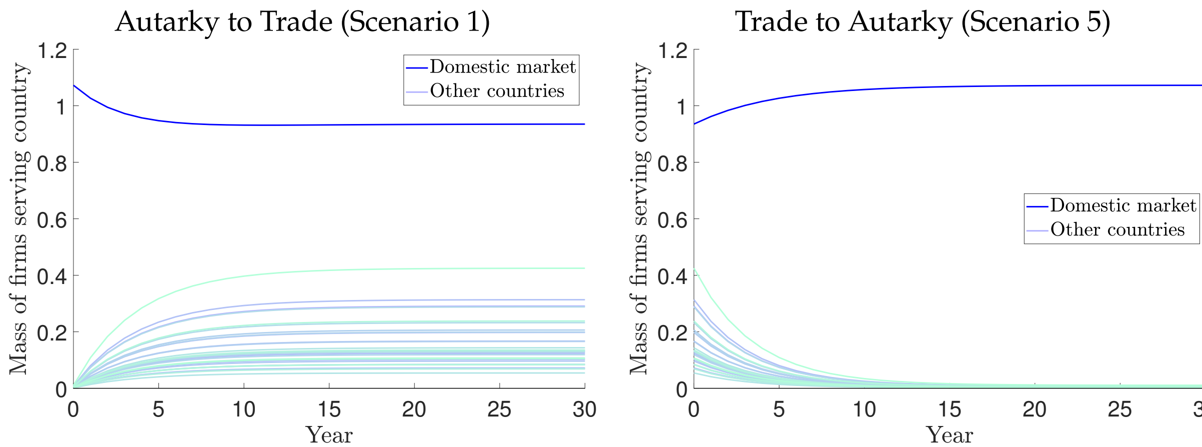
and then unexpectedly and permanently change trade costs κ_{ij} at time 1 to the value in the terminal steady state. When computing the welfare gains for country i, we make this adjustment to trade costs for all $j \neq i$ and leave trade costs for country pairs that do not involve i unchanged. To take changes in consumption over the transition path into account when computing the welfare gains, we first compute steady state consumption equivalents (see Appendix B.2 for details) and then take the ratio of these consumption equivalents.

Two conclusions stand out from the figure. First, the disparity between steady state gains and the full dynamic gains is relatively modest. On average in this sample, the autarky-to-trade gains are 15 percent smaller, and trade-to-autarky gains are 9 percent larger. Second, the steady state gains are always in between those two.

To illustrate the intuition for this ranking of gains, Appendix Figure C2 plots the dynamic paths of consumption, and Figure 4 plots the evolution of the masses of firms for one country, Malaysia.[20] When moving from autarky to trade, the country starts in the autarky steady state, and transitions to the trade steady state slowly. Over this transition, consumption is lower than in the terminal steady

[20] Appendix Figure C3 plots the consumption paths for Scenarios 1 and 5 for all countries.

Figure 4: Mass of Firms in the Transition Path, Malaysia



Notes: This figure shows the transition paths of the masses of Malaysian firms after a surprise permanent change in the trade regime. The left figure depicts the transition in Scenario 1 and the right figure the transition in Scenario 5. The dark line denotes the mass of Malaysian firms serving the domestic market n_{jjt} and the light lines denote the masses of Malaysian firms serving the other countries n_{jit}, $j \neq i$.

state. This is because agents need to invest in "exporting capital" n_{ijt}, $j \neq i$ starting from a level of 0, as illustrated in the left panel of Figure 4. Consumption only reaches the new steady state level once these masses of firms have converged to the higher level of the new steady state. As a result, the dynamic gains of going from autarky to trade are below the steady state comparison.

Moving from trade to autarky, countries' accumulated exporting capital n_{ijt} has become useless, because exports per unit mass of firms, x_{ijt}, are zero under infinite trade costs. At the same time, firms invest in their domestic operations, increasing the mass of domestic firms n_{iit} as shown in the right panel of Figure 4. The result is an immediate drop in consumption *below* the level of the autarky steady state, and a gradual convergence of consumption to the autarky steady state level from below (Appendix Figure C2). This reduces the consumption equivalent relative to the steady state – effectively the denominator of the GFT formula – and thus raises the implied GFT relative to the steady state comparison. The GFT numbers for each country and each scenario are listed in Appendix Table C1.

International asset markets and solo vs. simultaneous transitions. Table 2 investigates the importance of the international asset markets, and whether the country opens to trade alone or simultaneously with all other countries. It presents results for every combination of assumptions, listed in columns 3-4. Column 5 reports the mean GFT, and column 6 the correlation in GFT across countries between that row and the baseline (scenario 1).

Both assumptions make very little difference, delivering virtually identical GFT. The GFT are a tiny bit larger when other countries are also transitioning compared to a solo transition. Evidently, there is very limited impact of changes in firm creation in other countries on the consumption trajectory of the opening country. Similarly, whether during the transition the country has access to an international bond or is in financial autarky makes virtually no difference for the resulting GFT. Partial equilibrium reasoning might suggest that the introduction of the international bond expands the set of options available to agents, and thus should lead to higher welfare (since they can always choose not to trade the bond). However, this reasoning breaks down in general equilibrium environments. In general equilibrium, the introduction of an international bond changes all the prices in the economy, most notably the path of domestic interest rates. As a result, it is not clear that adding financial instruments will unambiguously increase all participants' welfare.

This is not to say that these assumptions are completely immaterial. Appendix Figure C2 plots the path of Malaysia's consumption following trade opening under the 5 scenarios. The blue and orange lines depict the consumption paths with international bonds, and the purple and yellow lines under financial autarky. Appendix Figure C4 shows the debt-to-GDP ratio of each opening country along its transition when all other countries are already open. The amount of borrowing is substantial, with debt-to-GDP ratio peaking at around 40% in the case of Malaysia. While there is no clear benchmark of how much a country transitioning out of autarky would be expected to borrow (as we never observe such a controlled scenario in practice), these amounts of debt are substantial relative to the observed total debt-to-GDP ratios of about 100% in advanced economies and 40% in emerging markets over the period 2000-present (International Monetary Fund, 2025). Thus, international borrowing enables the country to move consumption forward in a meaningful way. However, when a country borrows, it must repay, and its consumption stream remains below the steady state value for much longer compared to financial autarky. The net effect of the ability to borrow on life-time utility and hence steady state consumption equivalents following trade opening turns out to be negligible quantitatively. The lack of impact of financial integration echoes the finding of Gourinchas and Jeanne (2006) that the gains from financial integration are small when they come exclusively from speeding up the transition to the same steady state, as is the case in our exercise.

Relatedly, when international borrowing is allowed, the time profile of consumption depends meaningfully on whether other countries are opening at the same time (blue versus orange lines in Appendix Figure C2). When other countries are already open, the opening country can move consumption forward to a greater extent. This is sensible: when other countries are already in steady state, they do not need to undertake large investments in exporting capital required by trade opening. Thus, the solo opening country can borrow more and consume more earlier. By contrast, in financial autarky it makes no difference for the profile of consumption whether other countries are already open or not (in Appendix Figure C2, the purple and yellow lines are virtually indistinguishable from each other).

Robustness. We recompute the quantitative model with alternative parameter values to evaluate the robustness of our conclusions. As discussed above, the most important parameter is δ, governing the speed of the transition. We consider alternative values of $\delta = 0.35$ and $\delta = 0.15$. Additionally, we consider a higher demand elasticity of $\sigma = 1.5$. We further vary the curvature of entry costs χ,

Table 3: Robustness: Alternative Parameter Values

Calibration	Average Steady State Gains	Average Dynamic Gains: Autarky to Trade		Average Dynamic Gains: Trade to Autarky	
			difference		difference
	(1)	(2)	(3)	(4)	(5)
Baseline	0.641	0.547	-14.6%	0.698	9.0%
$\delta = 0.35$	0.641	0.570	-11.0%	0.683	6.6%
$\delta = 0.15$	0.641	0.500	-21.9%	0.730	13.9%
$\sigma = 1.5$	0.278	0.242	-12.8%	0.298	7.4%
$\chi = 1$	0.488	0.420	-14.0%	0.558	14.4%
$\chi = 0.3$	0.840	0.729	-13.2%	0.878	4.6%
$\beta = 0.94$	0.641	0.475	-25.8%	0.747	16.7%
$\gamma = 3$	0.641	0.542	-15.4%	0.702	9.5%

Notes: The table presents results from robustness checks that explore the implications of alternative calibrations. Column (1) shows the average gains from trade relative to autarky in steady state, using the formula in (3.4). Columns (2) and (3) show the absolute and relative gains in the quantitative dynamic Krugman model with transition when moving from autarky to trade (Scenario 1). Columns (4) and (5) show the absolute and relative gains in the dynamic Krugman model with transition when moving from trade to autarky (Scenario 5). The rows show alternative parameter choices. The first row shows the baseline calibration in Table 1. The next two rows change the speed of transition to a fast transition ($\delta = 0.35$) and a slow transition ($\delta = 0.15$). The next row changes the short-run iceberg elasticity σ. The subsequent two rows alter the convexity of adjustment χ. The second to last row explores the role of a lower discount factor β. The last row changes the relative risk aversion γ.

allowing for high curvature $\chi = 1$ and low curvature $\chi = 0.3$. Lastly, we show results for a lower discount factor $\beta = 0.94$ and a higher relative risk aversion $\gamma = 3$. The results are shown in Table 3. Across all calibrations, the steady state gains from trade implied by the formula in (3.4) remains a good approximation of quantitative gains from trade including transition dynamics, with average differences ranging from 4.6% to -25.8%. As expected, the largest average difference arises when households discount the future more ($\beta = 0.94$) and with a substantially slower transition ($\delta = 0.15$), and when moving from autarky to trade. Even here, the steady state gains from trade are a reasonable approximation. In all cases, the steady state gains from trade remain in between those computed in the full model going from autarky to trade and trade to autarky. We show the GFT numbers for each country and alternative parameter values in Appendix Tables C2 and C3.

6. Conclusion

Research employing dynamic trade and spatial models has exploded in recent years. We provide closed-form gains from trade formulas that apply in a wide class of such models with different microfoundations. In addition, we emphasize the important role of measurement. In a dynamic setting, care must be taken to obtain the elasticity required by the GFT formula from tariff elasticities. In our quantification the GFT are large, and the disparity with the static GFT formula can be substantial. Finally, we show that accounting for the transition path has a modest effect on the magnitude of the gains, even if countries can borrow and lend. Whether the steady-state formula over- or under-states the transition path gains depends on whether the transition is from autarky to trade or in the opposite direction.

In what sense are the gains from trade we study “dynamic”? Our goal in this paper is to analyze gains from trade in dynamic environments with forward-looking behavior and accumulation, where firms (or customers) adjust gradually in response to changes in trade costs. Because such models are analytically intractable in general, we approximate the gains from trade by comparing steady states and using insights from ACR. The key difference relative to the static models covered by ACR is that our class of models features endogenous adjustment in the mass of firms or customers, n_{ij}. This additional margin of adjustment alters the long-run relationship between trade costs and trade flows relative to static models, which changes the mapping between the long-run tariff and iceberg elasticities, and ultimately drives our main result. This difference is fully captured by parameter χ, the partial elasticity of n_{ij} with respect to x_{ij}. While our formula compares steady states, the steady states themselves embed dynamic accumulation decisions that are absent from standard static models. In this sense, the gains from trade we compute are “dynamic”: they reflect the long-run consequences of forward-looking entry and accumulation.

Our main quantitative result, that the gains from trade in dynamic trade models are large, is derived in models without extensions such as intermediate inputs, or multiple sectors. Costinot and Rodríguez-Clare (2014) quantify several of these extensions to the static ACR framework, and show they increase static gains from trade by as much as an order of magnitude. Incorporating such extensions would further amplify the already large GFT in dynamic settings and thus highlight the dramatic welfare impacts of changes in trade barriers.

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A. Theory Appendix

A.1 Proofs

Proof of Proposition 3.1. From A.1 and A.2, real consumption is proportional to the real wage:

$$C_j \propto \frac{w_j}{P_j}. \tag{A.1}$$

From A.3, the price index

$$P_j \propto w_j \lambda_{jj}^{-\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{\frac{1}{\varepsilon_\kappa^0}}. \tag{A.2}$$

From A.3, the mass of firms

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}}, \tag{A.3}$$

where we also used A.1. Putting (A.1)-(A.3) together yields the first result.

To derive the last claim, note that:

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} + \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}}.$$

It is immediate from A.3 that $\frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} = \chi$, and by assumption $\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} = \varepsilon_\kappa^0$ (also A.3). This gives the result.

Proof of Proposition 3.2. From A.1′,

$$Y_j = \left(1 - \frac{R_j^g}{Y_j}\right)^{-1} \left(w_j L_j + \Pi_j\right) \tag{A.4}$$

From A.2′,

$$C_j \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1} \frac{w_j}{P_j} \tag{A.5}$$

From A.3′,

$$\frac{w_j}{P_j} = \lambda_{jj}^{-\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{\frac{1}{\varepsilon_\kappa^0}} \tag{A.6}$$

Also from A.3′,

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}} \left(1 - \frac{R_j^g}{Y_j}\right)^{-\frac{\chi}{1+\chi}}, \tag{A.7}$$

Putting (A.5)-(A.7) together yields the first result. This last step also uses the fact that (A.4) and A.2′ imply that $\frac{Y_j}{w_j} \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1}$. The proof of the claim about the trade elasticity is identical to Proposition 3.1.

Proof of Proposition 3.3. Begin with combining $n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi}$ with $X_{ij} = n_{ij} x_{ij}$ both from A.3′ of Proposition 3.2 and the definition of expenditure shares $\lambda_{ij} = \frac{\tau_{ij} X_{ij}}{Y_j}$ to obtain

$$n_{ij} \propto \left(\frac{\frac{1}{\tau_{ij}} \lambda_{ij} Y_j}{w_i}\right)^{\frac{\chi}{1+\chi}}$$

for all i and j. Plugging this expression into the share (3.23) gives

$$\lambda_{ij} = \frac{\left(\lambda_{ij}\right)^{\frac{\chi}{1+\chi}} \left(\kappa_{ij}\right)^{\varepsilon_\kappa^0} \left(\tau_{ij} w_i\right)^{\varepsilon_\kappa^0 - \frac{\chi}{1+\chi}}}{\sum_k \left(\lambda_{kj}\right)^{\frac{\chi}{1+\chi}} \left(\kappa_{kj}\right)^{\varepsilon_\kappa^0} \left(\tau_{kj} w_k\right)^{\varepsilon_\kappa^0 - \frac{\chi}{1+\chi}}},$$

and solving for λ_{ij} yields

$$\lambda_{ij} = \frac{\left(\kappa_{ij}\right)^{\varepsilon_\kappa^0 (1+\chi)} \left(\tau_{ij} w_i\right)^{\varepsilon_\kappa^0 (1+\chi) - \chi}}{\left(\sum_k \left(\lambda_{kj}\right)^{\frac{\chi}{1+\chi}} \left(\kappa_{kj}\right)^{\varepsilon_\kappa^0} \left(\tau_{kj} w_k\right)^{\varepsilon_\kappa^0 - \frac{\chi}{1+\chi}}\right)^{(1+\chi)}}. \tag{A.8}$$

Now consider the expenditure share of j from ℓ, $\lambda_{\ell j}$, and sum over ℓ to get

$$\frac{\sum_\ell \left(\kappa_{\ell j}\right)^{\varepsilon_\kappa^0 (1+\chi)} \left(\tau_{\ell j} w_\ell\right)^{\varepsilon_\kappa^0 (1+\chi) - \chi}}{\left(\sum_k \left(\lambda_{kj}\right)^{\frac{\chi}{1+\chi}} \left(\kappa_{kj}\right)^{\varepsilon_\kappa^0} \left(\tau_{kj} w_k\right)^{\varepsilon_\kappa^0 - \frac{\chi}{1+\chi}}\right)^{(1+\chi)}} = \sum_\ell \lambda_{\ell j} = 1.$$

Using this relationship to substitute out the denominator in equation (A.8) gives

$$\lambda_{ij} = \frac{\left(\kappa_{ij}\right)^{\varepsilon_\kappa^0 (1+\chi)} \left(\tau_{ij} w_i\right)^{\varepsilon_\kappa^0 (1+\chi) - \chi}}{\sum_\ell \left(\kappa_{\ell j}\right)^{\varepsilon_\kappa^0 (1+\chi)} \left(\tau_{\ell j} w_\ell\right)^{\varepsilon_\kappa^0 (1+\chi) - \chi}}.$$

Since this relationship holds before and after the change in trade costs, we have, for $\hat{\lambda}_{ij} = \lambda'_{ij}/\lambda_{ij}$, that

$$\hat{\lambda}_{ij} = \frac{\left(\hat{\kappa}_{ij}\right)^{\varepsilon_\kappa^0 (1+\chi)} \left(\hat{\tau}_{ij} \hat{w}_i\right)^{\varepsilon_\kappa^0 (1+\chi) - \chi}}{\sum_\ell \lambda_{\ell j} \left(\hat{\kappa}_{\ell j}\right)^{\varepsilon_\kappa^0 (1+\chi)} \left(\hat{\tau}_{\ell j} \hat{w}_\ell\right)^{\varepsilon_\kappa^0 (1+\chi) - \chi}},$$

which is equation (3.24) in the proposition.

Next, note that tariff revenues can be written as

$$R_j^g = Y_j \left(1 - \sum_i \frac{1}{\tau_{ij}} \lambda_{ij}\right).$$

Plugging this into the budget constraint in A.1′ of Proposition 3.2 gives

$$Y_j = \frac{1}{\sum_i \frac{1}{\tau_{ij}} \lambda_{ij}} \left(w_j L_j + \Pi_j\right).$$

Now using A.2′ of Proposition 3.2 gives

$$Y_j \propto \frac{1}{\sum_k \frac{1}{\tau_{kj}} \lambda_{kj}} w_j L_j. \tag{A.9}$$

Next, combining trade balance from A.1′ of Proposition 3.2, $\sum_i X_{ij} = \sum_i X_{ji}$, with the definition of expenditure shares $\lambda_{ij} = \frac{\tau_{ij} X_{ij}}{Y_j}$, gives

$$Y_j \left(\sum_i \frac{1}{\tau_{ij}} \lambda_{ij}\right) = \sum_i \frac{1}{\tau_{ji}} \lambda_{ji} Y_i.$$

Combining this expression with (A.9) yields

$$w_j L_j = \sum_i \frac{\frac{1}{\tau_{ji}}\lambda_{ji}}{\sum_k \frac{1}{\tau_{ki}}\lambda_{ki}} w_i L_i.$$

Again, this relationship holds before and after the change in trade costs, so we have, for $\hat{w}_j = w_j'/w_j$, that

$$\hat{w}_j w_j L_j = \sum_i \frac{\frac{1}{\hat{\tau}_{ji}}\hat{\lambda}_{ji}\frac{1}{\tau_{ji}}\lambda_{ji}}{\sum_k \frac{1}{\hat{\tau}_{ki}}\hat{\lambda}_{ki}\frac{1}{\tau_{ki}}\lambda_{ki}} \hat{w}_i w_i L_i.$$

Now substituting for $\hat{\lambda}_{ji}$ and $\hat{\lambda}_{ki}$ using relationship (3.24) gives

$$\hat{w}_j = \sum_i \frac{\frac{1}{\tau_{ji}}\lambda_{ji}\left(\hat{\kappa}_{ji}\right)^{\varepsilon_\kappa^0(1+\chi)}\left(\hat{\tau}_{ji}\right)^{\left(\varepsilon_\kappa^0-1\right)(1+\chi)}\left(\hat{w}_j\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}\hat{w}_i \frac{w_i L_i}{w_j L_j}}{\sum_k \frac{1}{\tau_{ki}}\lambda_{ki}\left(\hat{\kappa}_{ki}\right)^{\varepsilon_\kappa^0(1+\chi)}\left(\hat{\tau}_{ki}\right)^{\left(\varepsilon_\kappa^0-\right)1(1+\chi)}\left(\hat{w}_k\right)^{\varepsilon_\kappa^0(1+\chi)-\chi}}.$$

This is equation (3.25) in the proposition.

A.2 Appendix Propositions

Proposition A.1. *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

A.1′ For all countries j, trade is balanced (expenditure = revenue):

$$P_j C_j = w_j L_j + \Pi_j + R_j^g$$

where

$$R_j^g = \sum_i \left(\tau_{ij} - 1\right) X_{ij}$$

and trade balance holds $\sum_i X_{ij} = \sum_i X_{ji}$.

A.2′ For all countries j, profits are a constant share of labor income:

$$\frac{\Pi_j}{w_j L_j} = const$$

A.3″ For all country pairs (i, j) trade flows satisfy

$$X_{ij} = n_{ij} x_{ij}$$

where

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^\chi$$

and domestic per-unit-mass sales satisfy

$$x_{jj} \propto \left(\frac{Y_j}{w_j}\right)^{\varepsilon^1} Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{A.10}$$

for some constants $\varepsilon^1 > 0$ and $\chi > 0$, and where $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij} / \partial \ln \kappa_{ij} < 0$ is the elasticity of exports per unit mass with respect to iceberg trade costs.

Then

$$C_j \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-\left(1 - \frac{1}{\varepsilon_\kappa^0}\frac{\chi}{1+\chi} - \frac{\varepsilon^1}{\varepsilon_\kappa^0}\right)} \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0}\frac{1}{1+\chi}} \tag{A.11}$$

where $\lambda_{jj} = \frac{X_{jj}}{Y_j}$, *and* $\varepsilon_\kappa^0(1+\chi)$ *is the long-run elasticity of trade flows with respect to iceberg trade costs.*

Proof. Derivations of (A.4) and (A.5) are identical to the steps in the proof of Proposition 3.2. From A.3″,

$$\frac{w_j}{P_j} = \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{-\frac{1}{\varepsilon_\kappa^0}} \left(\frac{Y_j}{w_j}\right)^{-\frac{\varepsilon^1}{\varepsilon_\kappa^0}}.$$

Also from A.3″,

$$n_{jj} \propto \left(\lambda_{jj}\frac{Y_j}{w_j}\right)^{\frac{\chi}{1+\chi}}.$$

Thus,

$$\frac{w_j}{P_j} \propto \left(\lambda_{jj}^{\frac{1}{1+\chi}} \left(\frac{Y_j}{w_j}\right)^{-\varepsilon^1 - \frac{\chi}{1+\chi}}\right)^{\frac{1}{\varepsilon_\kappa^0}}. \tag{A.12}$$

Putting (A.5) and (A.12) together yields the first result. This last step also uses the fact that (A.4) and A.2′ imply that $\frac{Y_j}{w_j} \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1}$. The proof of the claim about the trade elasticity is identical to Proposition 3.1.

□

Discussion. The conditions required for Proposition A.1 are identical to the conditions in Proposition 3.2 in every way except the per-firm sales (A.10). This functional form for sales is a strict generalization of (3.21), that allows per-firm sales to depend non-linearly on home market size and bilateral tariffs (recall from (A.4), Y_j is a function of total tariff revenue). The resulting gains from trade formula (A.11) differs from (3.22) by $\left(1 - \frac{R_j^g}{Y_j}\right)^{\frac{\varepsilon^1}{\varepsilon_\kappa^0}}$. Note that the alternative formulation for per-firm sales only affects the tariff adjustment component of the GFT formula. The non-tariff component is unchanged, and λ_{jj} is still raised to the power of the trade elasticity.

Proposition A.1 covers the Melitz (2003) model with tariffs. In that case, firm ω's sales are given by

$$x_{ijt}(\omega) = \frac{1}{\tau_{ijt}} C_{jt} \left(P_{jt}\right)^\sigma \left(\frac{\sigma}{\sigma-1}\frac{\tau_{ijt}\kappa_{ijt}}{\varphi(\omega)} w_{it}\right)^{1-\sigma}, \tag{A.13}$$

and the cutoff firm has productivity

$$\varphi_{ijt}^m = \frac{\sigma}{\sigma-1}\tau_{ijt}\kappa_{ijt}w_{it}\left(\frac{\sigma\tau_{ijt}w_{it}\xi_{ij}}{C_{jt}\left(P_{jt}\right)^\sigma}\right)^{\frac{1}{\sigma-1}}. \tag{A.14}$$

Combining these, the average firm sales are:

$$x_{ijt} \propto \left(\frac{1}{\tau_{ijt}}\frac{Y_{jt}}{w_{it}}\right)^{\frac{\theta}{\sigma-1}-1} \frac{1}{\tau_{ijt}} Y_{jt} \left(\frac{\tau_{ijt}\kappa_{ijt}w_{it}}{P_{jt}}\right)^{-\theta}. \tag{A.15}$$

Intuitively, tariffs and market size in the Melitz model affect the extensive margin, and thus appear non-linearly in the average firm sales. This property of the Melitz model with tariffs was pointed out by Felbermayr, Jung, and Larch (2015). It is easy to verify that the Melitz model with tariffs satisfies all the conditions for Proposition

A.1 to hold. As equation (A.15) makes clear, the Melitz model satisfies A.3″ for $\varepsilon^1 = \frac{\theta}{\sigma-1} - 1$.

What is notable about this functional form for ε^1 is that it goes to zero as $\frac{\theta}{\sigma-1} \to 1$. Di Giovanni, Levchenko, and Rancière (2011) and di Giovanni and Levchenko (2013) show that the distribution of sales to any destination in the Melitz-Pareto model follows a power law with exponent $-\frac{\theta}{\sigma-1}$. Further, these papers document that in the data, firm sales follow a power law with exponent close to -1, known as Zipf's Law (see also Axtell, 2001). This implies that when calibrated to the observed firm size distribution, $\frac{\theta}{\sigma-1} \approx 1$ and therefore $\varepsilon^1 \approx 0$. Intuitively, ε^1 appears because tariffs affect the extensive margin of exports conditional on drawing the sunk cost. As the firm size distribution approaches Zipf's Law, the extensive margin plays no role in the aggregate outcomes (see di Giovanni and Levchenko, 2013, for a detailed treatment of this result).

B. Quantitative Appendix

B.1 Equilibrium and Model Equations

For given sequences of trade costs $\left\{\kappa_{ijt}\right\}_{t=1}^{\infty}$ and tariffs $\left\{\tau_{ijt}\right\}_{t=1}^{\infty}$, as well as initial conditions n_{ij0} and $\frac{B^*_{j0}}{P_{US0}}$, an equilibrium of this economy are sequences of individual trade flows $\left\{\frac{x_{ijt}}{P_{it}}\right\}_{t=1}^{\infty}$ (J^2 variables), aggregate trade flows $\left\{\frac{X_{ijt}}{P_{it}}\right\}_{t=1}^{\infty}$ (J^2 variables), masses of firms $\left\{n_{ijt}\right\}_{t=1}^{\infty}$ (J^2 variables), firm values $\left\{\frac{v_{ijt}}{P_{it}}\right\}_{t=1}^{\infty}$ (J^2 variables), real wages $\left\{\frac{w_{it}}{P_{it}}\right\}_{t=1}^{\infty}$ (J variables), consumption $\{C_{it}\}_{t=1}^{\infty}$ (J variables), real exchange rates $\left\{\frac{P_{it}}{P_{USt}}\right\}_{t=1}^{\infty}$ ($J-1$ variables), bond holdings $\left\{\frac{B^*_{jt}}{P_{USt}}\right\}_{t=1}^{\infty}$ (J variables), and the interest rate $\left\{1+r^*_t\right\}_{t=1}^{\infty}$ (1 variable) such that the following equations hold:

- Current account ($J-1$ equations, one is redundant by Walras' law):

$$\frac{B^*_{jt}}{P_{USt}} - \left(1+r^*_{t-1}\right)\frac{B^*_{jt-1}}{P_{USt-1}} = \frac{P_{jt}}{P_{USt}}\sum_{i=1}^{J}\frac{X_{jit}}{P_{jt}} - \sum_{i=1}^{J}\frac{P_{it}}{P_{USt}}\frac{X_{ijt}}{P_{it}}$$

- Euler equation of internationally-traded bond (J equations):

$$C_{jt}^{-\gamma} = \frac{1+r^*_t}{1+\frac{w_{jt}}{P_{jt}}\psi'\left(\left(\frac{P_{jt}}{P_{USt}}\right)^{-1}\frac{B^*_{jt}}{P_{USt}}\right)}\frac{\frac{P_{jt}}{P_{USt}}}{\frac{P_{jt+1}}{P_{USt+1}}}\beta C_{jt+1}^{-\gamma}$$

- Price index (J equations):

$$1 = \left(\sum_{i} n_{ijt-1}\left(\frac{\sigma}{\sigma-1}\tau_{ijt}\kappa_{ijt}\frac{\frac{P_{it}}{P_{USt}}}{\frac{P_{jt}}{P_{USt}}}\frac{w_{it}}{P_{it}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

- Mass of traded goods (J^2 equations):

$$n_{ijt} = (1-\delta)\,n_{ijt-1} + \left(b\frac{\frac{v_{ijt}}{P_{it}}}{\frac{w_{it}}{P_{it}}}\right)^{\chi}$$

- Value of firms (J^2 equations):

$$\frac{v_{ijt}}{P_{it}} = \beta\frac{C_{it+1}^{-\gamma}}{C_{it}^{-\gamma}}\left(\frac{1}{\sigma}\frac{x_{ijt+1}}{P_{it+1}} + (1-\delta)\frac{v_{ijt+1}}{P_{it+1}}\right)$$

- Labor market clearing (J equations):

$$L_i = \frac{\sigma-1}{\sigma}\frac{1}{\frac{w_{it}}{P_{it}}}\sum_{j=1}^{J} n_{ijt-1}\frac{x_{ijt}}{P_{it}} + b^{\chi}\frac{\chi}{\chi+1}\sum_{j=1}^{J}\left(\frac{\frac{v_{ijt}}{P_{it}}}{\frac{w_{it}}{P_{it}}}\right)^{\chi+1} + \psi\left(\frac{1}{\frac{P_{it}}{P_{USt}}}\frac{B^*_{it}}{P_{USt}}\right)$$

- Aggregate trade flows (J^2 equations):

$$\frac{X_{ijt}}{P_{it}} = n_{ijt-1}\frac{x_{ijt}}{P_{it}}$$

- Individual trade flows (J^2 equations):

$$\frac{x_{ijt}}{P_{it}} = \frac{1}{\tau_{ijt}} C_{jt} \left(\frac{\frac{P_{jt}}{P_{USt}}}{\frac{P_{it}}{P_{USt}}} \right)^{\sigma} \left(\frac{\sigma}{\sigma - 1} \tau_{ijt} \kappa_{ijt} \frac{w_{it}}{P_{it}} \right)^{1-\sigma}$$

- Market clearing for international bond (1 equation):

$$\sum_{j=1}^{J} \frac{B^*_{jt}}{P_{US,t}} = 0$$

and that all households' transversality conditions are satisfied.

B.2 Dynamic Transitions

This section details the procedure to compute the dynamic welfare gains presented in Figure 3 and Table C1, among others. We use the 30-country calibration listed in Table C1 and compute the five different scenarios listed in Table 2.

We first compute the steady states of the model under trade and under autarky. The steady state under trade matches the observed expenditure shares and tariffs for 2006. The steady state under autarky depends on the scenario and is computed as discussed in Section 5.3. In all scenarios, we consider an unexpected permanent shock to the non-tariff trade costs in period 1.

We use the Newton algorithm in order to compute the transition path of the model variables, where period 0 represents the initial steady state. Depending on the scenario, convergence to the terminal steady state requires computing the transition path for up to 4000 periods. All parameters other than a particular set of non-tariff iceberg trade costs κ_{ij} (which depend on the scenario) remain constant throughout the simulations.

We base the gains from trade calculations over the transition path on consumption equivalent variation. The present value of consumption of the representative household in country j in period 1 is

$$V_{j1} = \sum_{t=1}^{\infty} \beta^{t-1} \frac{\left(C_{jt}\right)^{1-\gamma}}{1-\gamma}.$$

Autarky to trade. Consider the transition path from autarky to trade. Let the superscript T denote the transition path under trade and superscript A denote the initial steady state under autarky. We then compute the present value of consumption under the transition path to trade as

$$V^T_{j1} = \sum_{t=1}^{\infty} \beta^{t-1} \frac{\left(C^T_{jt}\right)^{1-\gamma}}{1-\gamma}.$$

Now, assume a case where the household receives a constant consumption equivalent $C^{T,e}_j$ in every period, such that

$$V^{T,e}_{j1} = \sum_{t=1}^{\infty} \beta^{t-1} \frac{\left(C^{T,e}_{j}\right)^{1-\gamma}}{1-\gamma},$$

where the superscript e denotes the consumption equivalent.

Setting $V_{j1}^{T} = V_{j1}^{T,e}$ gives

$$C_j^{T,e} = \left((1-\beta) \sum_{t=1}^{\infty} \beta^{t-1} \left(C_{jt}^{T} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}},$$

which is our measure of welfare associated with the transition path to trade. The dynamic gains from trade for a scenario involving the transition from autarky to trade are defined as

$$DGFT_j^{A \to T} = \frac{C_j^{T,e}}{C_j^{A}}.$$

Trade to autarky. We also analyze the transition path from trade to autarky. In this case, the superscript A denotes the transition path under autarky and superscript T denotes the initial steady state under trade. We compute the present value of consumption under the autarky transition path as

$$V_{j1}^{A} = \sum_{t=1}^{\infty} \beta^{t-1} \frac{\left(C_{jt}^{A} \right)^{1-\gamma}}{1-\gamma}.$$

Following similar steps as above, the welfare measure associated with the transition to autarky is

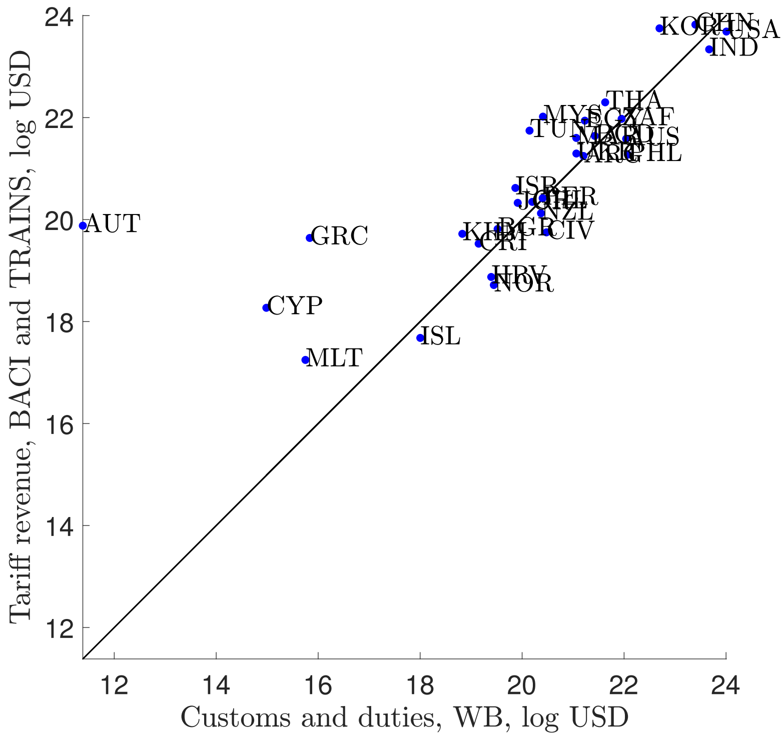
$$C_j^{A,e} = \left((1-\beta) \sum_{t=1}^{\infty} \beta^{t-1} \left(C_{jt}^{A} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

The dynamic gains from trade are then

$$DGFT_j^{T \to A} = \frac{C_j^{T}}{C_j^{A,e}}.$$

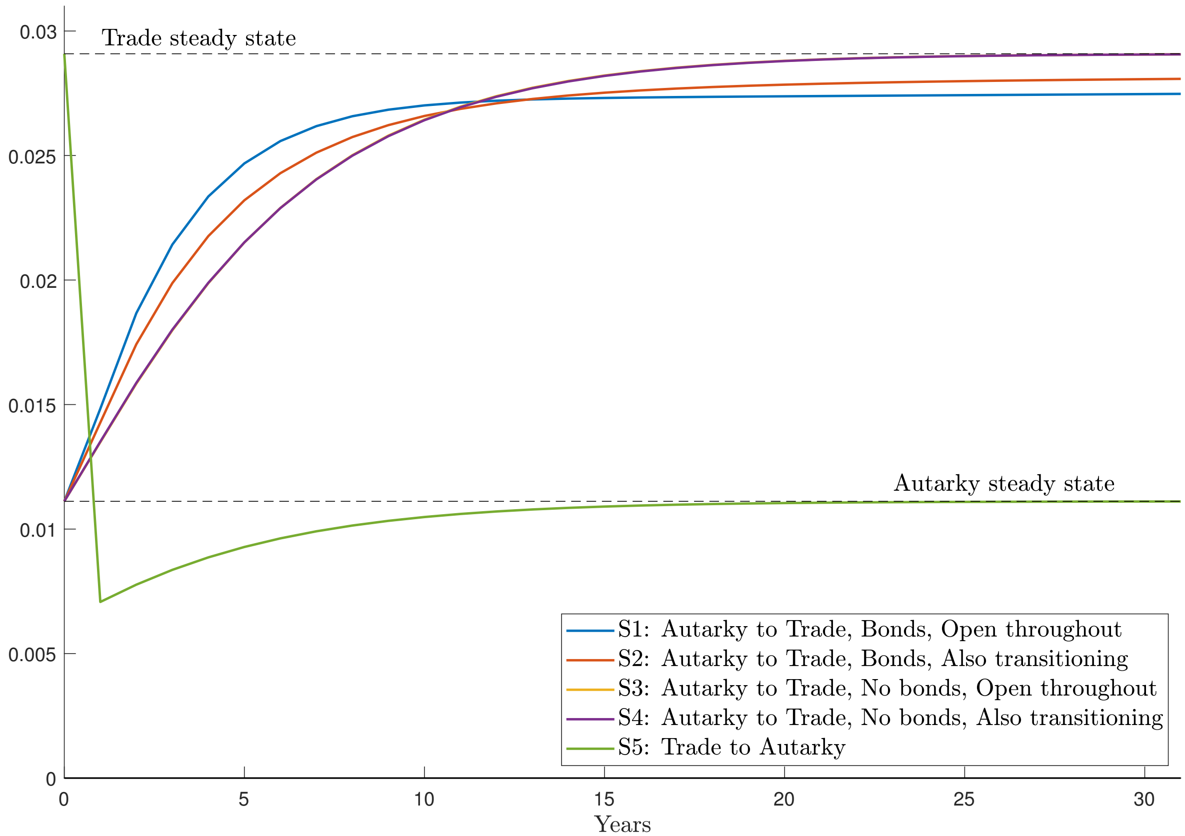
C. Additional Results

Figure C1: Tariff Revenue Comparison



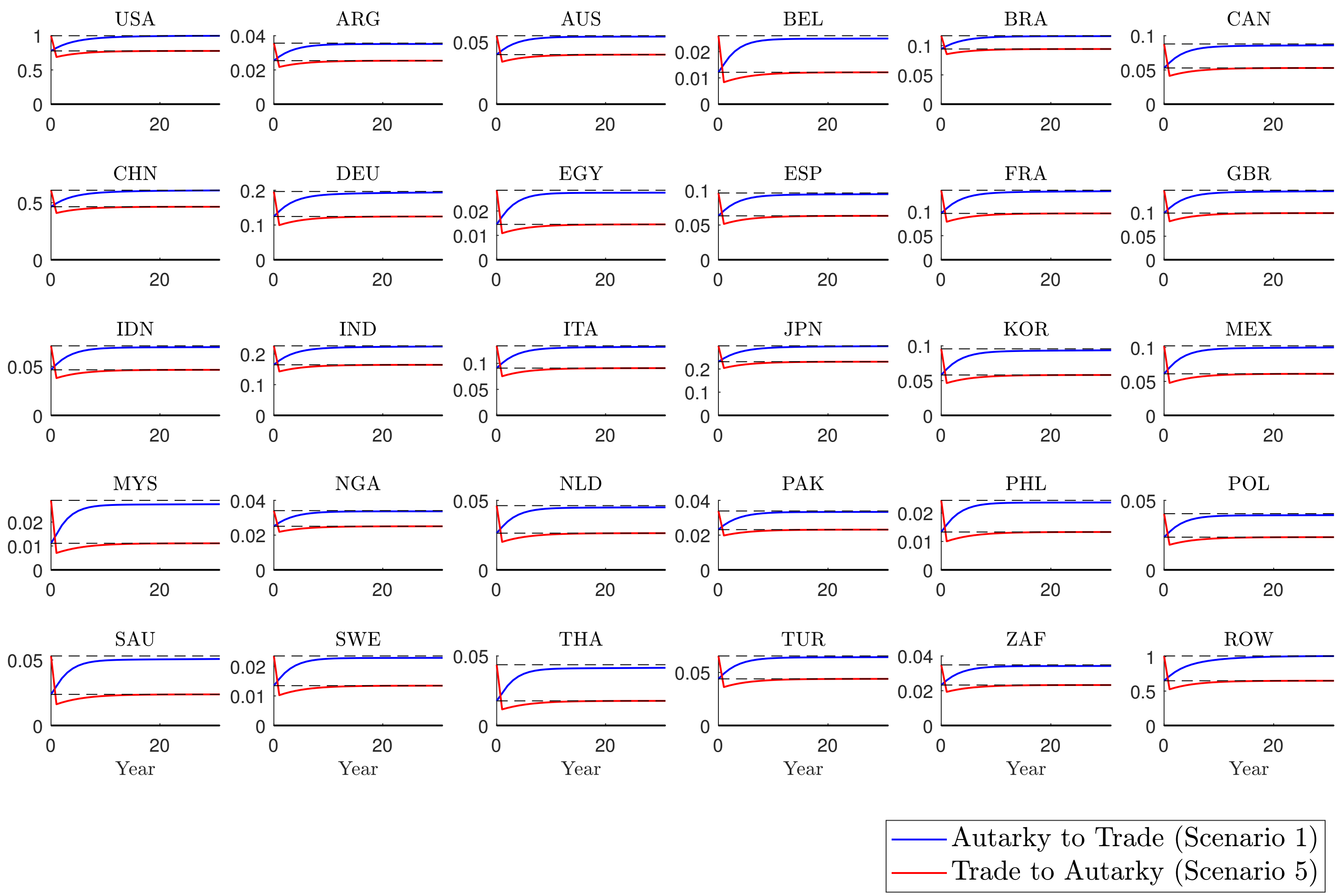
Notes: The figure compares the log of tariff revenues calculated from BACI-TRAINS with the log of customs and duties from the World Bank (WB) for the year 2006.

Figure C2: Consumption Paths in Different Scenarios: Malaysia



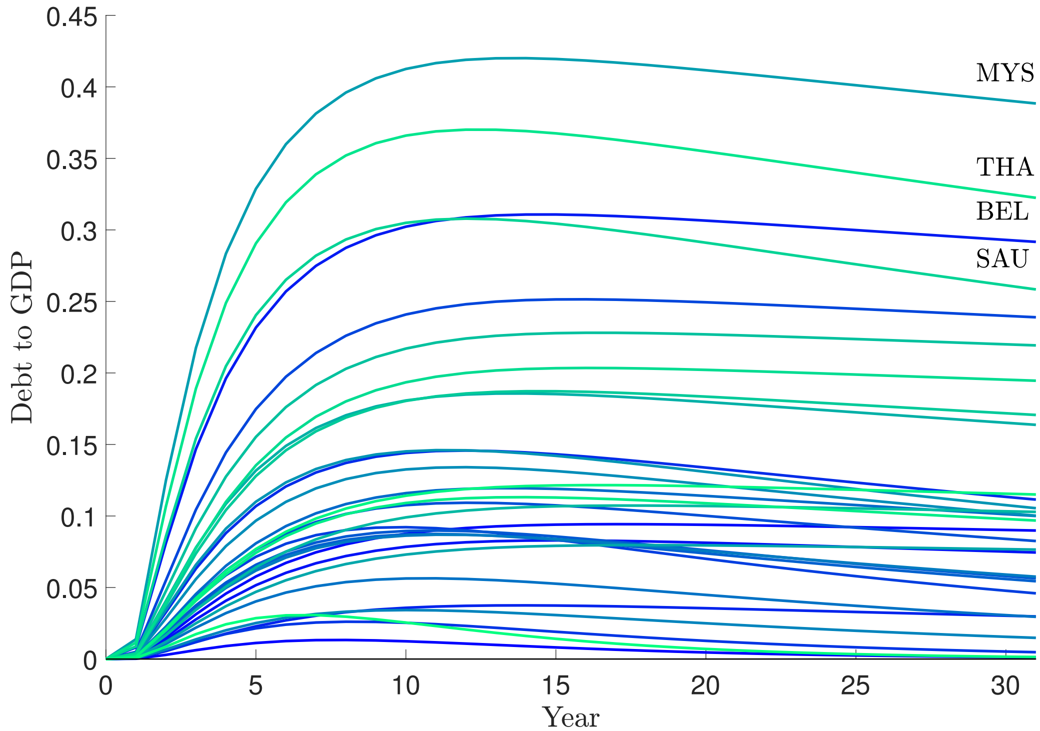
Notes: This figure shows the consumption paths following trade opening and closing under the different scenarios summarized in Table 2.

Figure C3: Consumption along the Transition Paths



Notes: The figure shows the transition paths of consumption for all countries included in our baseline calibration. Each plot shows two different scenarios, Scenario 1 and 5. In Scenario 1 only the country in the title of the subplot transitions. The shock is unanticipated and occurs in period 1. Dashed lines mark the steady state values under trade and autarky. Consumption is normalized to one for the USA in the trade steady state.

Figure C4: Transition Paths of Debt-to-GDP Ratios



Notes: The figure shows the debt-to-GDP ratios for all countries included in our baseline calibration along the transition from autarky to trade (Scenario 1). The shock is unanticipated and occurs in period 1.

Table C1: Net Dynamic Gains from Trade by Country

Country	Steady state comp.	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
MYS	1.617	1.336	1.326	1.321	1.322	1.796
THA	1.469	1.220	1.212	1.209	1.209	1.626
SAU	1.236	1.035	1.030	1.028	1.028	1.362
BEL	1.149	0.967	0.961	0.960	0.960	1.268
EGY	0.966	0.819	0.815	0.814	0.814	1.047
PHL	0.835	0.712	0.709	0.709	0.709	0.910
NLD	0.750	0.642	0.640	0.640	0.640	0.819
SWE	0.747	0.639	0.637	0.637	0.637	0.815
POL	0.717	0.615	0.613	0.613	0.613	0.783
MEX	0.673	0.578	0.577	0.577	0.577	0.732
CAN	0.663	0.570	0.569	0.569	0.569	0.722
KOR	0.647	0.556	0.555	0.555	0.555	0.699
DEU	0.578	0.499	0.498	0.498	0.498	0.628
ROW	0.555	0.479	0.478	0.478	0.478	0.601
IDN	0.526	0.456	0.455	0.455	0.455	0.570
ESP	0.520	0.450	0.449	0.449	0.449	0.564
FRA	0.500	0.433	0.433	0.433	0.433	0.542
ZAF	0.497	0.431	0.430	0.430	0.430	0.536
TUR	0.491	0.426	0.425	0.425	0.425	0.532
GBR	0.490	0.424	0.424	0.424	0.424	0.530
ITA	0.472	0.409	0.409	0.409	0.409	0.511
PAK	0.463	0.402	0.401	0.401	0.401	0.495
ARG	0.404	0.352	0.352	0.352	0.352	0.435
AUS	0.382	0.333	0.333	0.333	0.332	0.411
IND	0.376	0.328	0.328	0.327	0.327	0.403
NGA	0.359	0.314	0.314	0.314	0.314	0.385
CHN	0.312	0.273	0.273	0.273	0.273	0.335
JPN	0.296	0.259	0.259	0.259	0.259	0.318
USA	0.285	0.250	0.250	0.250	0.250	0.307
BRA	0.240	0.212	0.212	0.211	0.211	0.257

Notes: The table presents the net dynamic GFT underlying Table 2. Steady state gains are based on equation (3.22). For details in the scenarios, see Section 5.3 and Appendix B.2. Countries are ordered in descending order of their GFT.

Table C2: Net Dynamic Gains from Trade by Country, Autarky to Trade, Robustness

Country	Steady state comp.	$\delta = 0.35$	$\delta = 0.15$	$\sigma = 1.5$	$\chi = 1$	$\chi = 0.3$	$\beta = 0.94$	$\gamma = 3$
MYS	1.617	1.403	1.202	0.537	0.989	1.869	1.131	1.314
THA	1.469	1.280	1.100	0.498	0.908	1.692	1.037	1.201
SAU	1.236	1.084	0.937	0.432	0.776	1.422	0.886	1.021
BEL	1.149	1.011	0.877	0.405	0.721	1.330	0.829	0.954
EGY	0.966	0.855	0.746	0.360	0.640	1.077	0.706	0.810
PHL	0.835	0.743	0.651	0.313	0.546	0.949	0.617	0.706
NLD	0.750	0.669	0.587	0.283	0.489	0.859	0.558	0.636
SWE	0.747	0.666	0.585	0.281	0.485	0.857	0.556	0.634
POL	0.717	0.641	0.563	0.271	0.468	0.823	0.535	0.610
MEX	0.673	0.602	0.529	0.258	0.443	0.767	0.504	0.573
CAN	0.663	0.593	0.522	0.254	0.436	0.758	0.497	0.565
KOR	0.647	0.579	0.510	0.253	0.435	0.725	0.485	0.552
DEU	0.578	0.519	0.458	0.225	0.382	0.660	0.436	0.494
ROW	0.555	0.498	0.439	0.217	0.369	0.630	0.419	0.473
IDN	0.526	0.473	0.419	0.208	0.353	0.596	0.399	0.453
ESP	0.520	0.468	0.414	0.204	0.346	0.593	0.395	0.447
FRA	0.500	0.450	0.398	0.197	0.334	0.570	0.380	0.430
ZAF	0.497	0.447	0.396	0.199	0.338	0.558	0.378	0.428
TUR	0.491	0.443	0.392	0.195	0.329	0.558	0.374	0.423
GBR	0.490	0.441	0.390	0.194	0.327	0.557	0.373	0.421
ITA	0.472	0.425	0.377	0.187	0.316	0.537	0.360	0.406
PAK	0.463	0.417	0.370	0.190	0.324	0.509	0.353	0.400
ARG	0.404	0.365	0.325	0.165	0.277	0.453	0.310	0.350
AUS	0.382	0.345	0.307	0.155	0.259	0.432	0.294	0.331
IND	0.376	0.340	0.302	0.155	0.261	0.417	0.289	0.326
NGA	0.359	0.326	0.290	0.149	0.250	0.399	0.277	0.312
CHN	0.312	0.283	0.252	0.130	0.216	0.349	0.242	0.272
JPN	0.296	0.269	0.240	0.122	0.203	0.335	0.230	0.258
USA	0.285	0.259	0.232	0.118	0.196	0.323	0.222	0.249
BRA	0.240	0.219	0.196	0.102	0.169	0.268	0.188	0.211

Notes: The table presents the net dynamic GFT underlying Table 3. Steady state gains are based on equation (3.22). For details in the scenarios, see Section 5.3 and Appendix B.2. Countries are ordered in descending order of their GFT.

Table C3: Net Dynamic Gains from Trade, Trade to Autarky, Robustness

Country	Steady state comp.	$\delta = 0.35$	$\delta = 0.15$	$\sigma = 1.5$	$\chi = 1$	$\chi = 0.3$	$\beta = 0.94$	$\gamma = 3$
MYS	1.617	1.750	1.894	0.677	1.388	2.359	1.951	1.815
THA	1.469	1.586	1.713	0.626	1.264	2.121	1.762	1.642
SAU	1.236	1.329	1.432	0.540	1.066	1.760	1.471	1.373
BEL	1.149	1.237	1.333	0.506	0.989	1.640	1.370	1.278
EGY	0.966	1.026	1.092	0.443	0.849	1.304	1.118	1.053
PHL	0.835	0.890	0.951	0.386	0.728	1.145	0.974	0.915
NLD	0.750	0.801	0.857	0.349	0.652	1.034	0.878	0.823
SWE	0.747	0.797	0.853	0.347	0.648	1.031	0.874	0.820
POL	0.717	0.766	0.819	0.335	0.623	0.988	0.839	0.787
MEX	0.673	0.717	0.765	0.317	0.587	0.918	0.783	0.736
CAN	0.663	0.707	0.754	0.313	0.577	0.906	0.772	0.725
KOR	0.647	0.686	0.728	0.309	0.569	0.863	0.744	0.702
DEU	0.578	0.615	0.656	0.276	0.504	0.784	0.671	0.630
ROW	0.555	0.589	0.627	0.267	0.485	0.747	0.641	0.603
IDN	0.526	0.558	0.593	0.254	0.461	0.705	0.606	0.571
ESP	0.520	0.552	0.588	0.251	0.454	0.701	0.601	0.566
FRA	0.500	0.531	0.565	0.242	0.437	0.674	0.578	0.544
ZAF	0.497	0.526	0.557	0.242	0.438	0.658	0.569	0.537
TUR	0.491	0.521	0.554	0.238	0.430	0.659	0.567	0.534
GBR	0.490	0.520	0.553	0.237	0.428	0.658	0.565	0.532
ITA	0.472	0.501	0.532	0.229	0.412	0.633	0.544	0.512
PAK	0.463	0.487	0.513	0.230	0.413	0.596	0.523	0.496
ARG	0.404	0.427	0.451	0.200	0.357	0.530	0.461	0.436
AUS	0.382	0.404	0.428	0.189	0.335	0.505	0.437	0.412
IND	0.376	0.395	0.417	0.188	0.333	0.487	0.425	0.403
NGA	0.359	0.378	0.399	0.180	0.319	0.465	0.407	0.386
CHN	0.312	0.329	0.347	0.157	0.276	0.406	0.354	0.335
JPN	0.296	0.313	0.331	0.149	0.260	0.389	0.338	0.319
USA	0.285	0.301	0.319	0.144	0.251	0.375	0.325	0.308
BRA	0.240	0.253	0.267	0.123	0.213	0.310	0.272	0.258

Notes: The table presents the net dynamic GFT underlying Table 3. Steady state gains are based on equation (3.22). For details in the scenarios, see Section 5.3 and Appendix B.2. Countries are ordered in descending order of their GFT.